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EVALUATING SCIENTIFIC PRODUCTS BY MEANS OF CITATION-BASED MODELS: A FIRST ANALYSIS AND VALIDATION

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Abstract. Some integrated models for ranking scientific publications together with authors and journals are presented and analyzed. The models rely on certain adiacency matrices obtained from the relations of citation, authorship and publication, which concurr to forming a suitable irreducible stochastic matrix whose Perron vector provides the ranking. Some perturbation theorems concerning the Perron vector of nonnegative irreducible matrices are proved. These theoretical results provide a validation of the consistency and effectiveness of our models. Several paradigmatic examples are reported together with some results obtained on a real set of data.

Key words. Page Rank, Perron vector, perturbation results, impact factor

AMS subject classifications. 65F15

1. Introduction. Ranking scientific publications independently of their contents is a problem of great practical importance and of particular theoretical interest. Most of the attempts for evaluating the quality of a scientific publication are based on the analysis of the citations received.

Recently, a certain interest has been addressed to citation analysis and to the related models, mainly because it enables one to rigorously measure delicate concepts that otherwise would be difficult to capture, as the quality of the research performed by scholars or the reputation and the influence of researchers. Indeed, only a careful reading of a paper can tell one which is the real nature of a citation; in fact, an analysis independent of the context cannot distinguish between critic and positive citations. However, it is interesting to point out that in all the models presented in the literature, receiving a citation is considered a positive fact whatever is the nature of the citation.

A common measure to assess the importance of a scientific journal is the well known *Impact Factor* calculated by the Institute of Scientific Information (ISI) and introduced by Garfield [8]. However, not all the scientific community agrees about the effectiveness of such a measure, because regarding all the citations with the same weight, is essentially a metric of popularity and it seems not to capture criteria such as prestige, or importance [4]. Many other proposals have been done over the years starting with the one of Pinski and Narin [15] where the authors anticipated of many years the Google model [5]. This same model has been recently reconsidered [14] and it has been proved that this kind of approach is the only one satisfying a number of very reasonable requirements. Another proposal is the *Eigenfactor* method [2] that combines a Google-like approach with a time aware mechanism.

Most of the related literature addresses the problem of ranking journals [4, 14, 15], some other authors proposed strategies for ranking scholars [10, 13] and scientific institutions [16, 17]. In our study we aim to present and analyze an integrated model where more subjects and their relationships are considered, like authors, papers, journals, fields and institutions.

In particular, the idea is that for determining the importance of a journal one has

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to take into account not only the "quality " of the citations from other papers (as done by the ranking schemes in the literature), but also the "quality" of papers and of their authors. Similarly, an author is important if he/she publishes important papers on important journals and maybe with important co-authors. A paper is important if it receives citations from other important papers, but also if it is published on important journals and is written by important authors. This leads to an integrated model where each player —journals, authors and papers— contributes in the determination of the score of the others. Throughout, we refer to these players with the term *subjects*.

The basic principle that we follow, is that the importance of a subject is the weighted sum of the importances of all the subjects that are related to it in a sense that will be made clear later on. In this model, the sum of the weight coefficients must be one so that the overall amount of importance is neither destroyed nor created.

We start with the simple *one-class model* where only the class of *Papers* is taken into account, and where the importance is given on the basis of *citations*. Then we consider more general models where other actors are involved. The *two-class model* takes into account the class of *Authors* besides that of Papers and the importance is given on the basis of citations and of *authorship*. The *three-class model* adds to the latter the class of *Journals*. More elaborated models involving, say, research areas and institutions can be in principle introduced and are left to a future work.

In all these models, the vector with the rating of all the involved subjects is obtained as the positive invariant vector of an irreducible row-stochastic matrix, normalized so that the sum of its components is one (Perron vector).

We perform a consistency analysis of the introduced models and prove new perturbation theorems concerning the Perron vector of the stochastic matrices involved which extend some result given in [6]. These perturbation results are the matrix formulation of the desired properties which are consistent with our models. In particular, in the one-class model, we prove that a paper which receives a new citation has an increasing of its rank which is larger than the increasing received by the other papers. Similarly, we prove that if a new paper is introduced and this paper contains a citation to a given paper, then the importance of the latter has an increasing larger than the ones received by the other papers. These properties keep their validity in the two-class and in the three-class models. Several examples are given which confirm the expected properties.

The paper is organized as follows: in Section 2 we introduce and analyze the one-class model; in Section 3 we describe the two-class model and its modifications; Section 3 contains a brief description of the three-class model. In Section 4 we report on the results of some experiments performed on the basis of the CiteSeer.IST database while in Section 5 we draw conclusions and discuss some open issues.

2. One-class model. Assume we are given n papers numbered from 1 to n together with the $n \times n$ adjacency matrix $H = (h_{i,j})$ such that $h_{i,j} = 1$ if paper i cites paper j, $h_{i,j} = 0$ otherwise. Following a model similar to Google [5] we assume that the importance p_j of paper j is given by the importances of the papers p_i that cite paper j, scaled by the factor d_i which is the number of citations contained in paper i. In this way, the importance given by paper i is uniformly distributed among all the papers cited therein, and the principle that the importance of a subject is neither destroyed nor created is respected.

Here and hereafter, we denote by $\boldsymbol{e}$ the vector of appropriate length with all components equal to 1. We denote by $\boldsymbol{e}_k$ the k-th column of the identity matrix of appropriate size. The size of vectors and matrices, if not specified, is deduced by the

context. Given a vector $\boldsymbol{v} = (v_i)$ of n components, with the expression $\mathrm{diag}(\boldsymbol{v})$ we denote the $n \times n$ diagonal matrix having diagonal entries v_i, $i = 1, \ldots, n$.

The scaling factors $d_i = \sum_j h_{i,j}$ define the vector $\boldsymbol{d} = (d_i)$ which satisfies the equation $\boldsymbol{d} = H\boldsymbol{e}$. Moreover, if $d_i \neq 0$ for any i, the matrix

$$P = (p_{i,j}) = \mathrm{diag}(\boldsymbol{d})^{-1} H$$

is row-stochastic, that is $\sum_j p_{i,j} = 1$.

Since in principle there might be papers with an empty set of citations, the matrix H might have some null row and some factor d_i might be zero. This fact may make the model inconsistent. We cure this drawback by introducing a *dummy paper*, paper $n+1$, which cites and is cited by all the existing papers except itself. In this way the new adjacency matrix of size $n+1$, which with an abuse of notation we still denote by H, has no null row and is irreducible. From the modeling point of view, the dummy paper collects the importance of all the papers and redistributes it uniformly to all the subjects.

Note that the introduction of the dummy paper guarantees that the matrix P is stochastic, acyclic and aperiodic. This provides important computational advantages in the numerical solution of the model. It is interesting to observe that a similar technique is used in the Google model where, unlike in our case, a damping factor is also introduced.

The equation that we obtain in this way is

$$\boldsymbol{p}^T = \boldsymbol{p}^T P, \qquad P = \mathrm{diag}(H\boldsymbol{e})^{-1} H \tag{2.1}$$

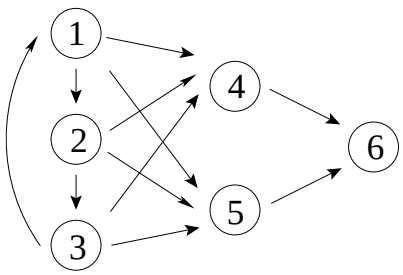
and, since the matrix $\mathrm{diag}(H\boldsymbol{e})^{-1}H$ is nonnegative and irreducible, from the Perron-Frobenius theorem there exists unique a vector $\boldsymbol{p} = (p_i)$ such that $p_i > 0$, $\sum_i p_i = 1$, which solves (2.1). We call $\boldsymbol{p}$ the *Perron vector* of P.

Equation (2.1) states that the importance of paper j is given by the sum of the importances received by all the other papers, that is, by the values p_i scaled by the factors $h_{i,j}/\sum_s h_{i,s}$, $i = 1, \ldots, n+1$, i.e.,

$$p_j = \sum_{i=1}^{n+1} p_i \frac{h_{i,j}}{\sum_{s=1}^{n+1} h_{i,s}}, \qquad j = 1, 2, \ldots, n = 1.$$

In fact, each paper i uniformly distributes its importance to all the $\sum_s h_{i,s}$ papers that it cites.

Example 1. Consider the case of 6 papers where citations are given by the following graph where we have not reported the node corresponding to the dummy paper.



The adjacency matrix, including the dummy paper, is

$$H = \left[\begin{array}{cccccc|c} 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{array}\right].$$

Papers 1,2, and 3 are on the same rank level: except for the dummy paper, they receive one citation each and are inside a cycle. Papers 4 and 5 receive three citations by papers 1,2,3 and are on the same level but in a higher position with respect to papers 1,2, and 3. Paper 6 receives only two citations by papers 4 and 5. Therefore, in a model based only on the number of citations, the rank of paper 6 should be inferior to the rank of papers 4 and 5. However, since paper 6 is cited by two papers which are more important than papers 1,2, and 3, one should expect that in our model its rank is higher. In fact, the left eigenvector of $\mathrm{diag}(H\boldsymbol{e})^{-1}H$ is

$$\boldsymbol{p}^T = (0.0784314,\ 0.0784314,\ 0.0784314,\ 0.117647,\ 0.117647,\ 0.176470,\ 0.352941)$$

where $p_1 = p_2 = p_3 < p_4 = p_5 < p_6$ and paper 6 reaches the highest rank as expected (the dummy paper gets always a greater score). Modifying the data by adding a citation from paper 5 to paper 4 yields the vector

$$\boldsymbol{p}^T = (0.075472,\ 0.075472,\ 0.075472\, 0.150943,\ 0.113208,\ 0.169811,\ 0.339623)$$

where $p_1 = p_2 = p_3 < p_5 < p_6 < p_4$ and paper 4 reaches the highest rank. □

An interesting question is to figure out what happens of the Perron vector $\boldsymbol{p}$ of the matrix P if P is perturbed in the following way: a new link is inserted in the graph connecting node r to node s where in the original adjacency matrix $h_{r,s} = 0$. That is, the new matrix $\widehat{H}$ is constructed in such a way that $\widehat{h}_{r,s} = 1$ while $\widehat{h}_{i,j} = h_{i,j}$ for the remaining entries and $\widehat{P} = \mathrm{diag}(\widehat{H}\boldsymbol{e})^{-1}\widehat{H}$.

One would expect that the paper receiving the new citation increases its value more than the other papers do, i.e., the component $\widehat{p}_s$ of the Perron vector $\widehat{\boldsymbol{p}}$ of the matrix $\widehat{P}$ constructed from $\widehat{H}$ obtains a larger increase with respect to the remaining components. Formally, $\widehat{\boldsymbol{p}}_s/\boldsymbol{p}_s \geq \widehat{\boldsymbol{p}}_i/\boldsymbol{p}_i$ for any i.

The following result of [6], which extend the result of [7], is useful for providing a formal proof of this fact.

THEOREM 2.1. *Let A and B be $n \times n$ nonnegative irreducible matrices having the same spectral radius ρ. Let $\boldsymbol{x} = (x_i)$ and $\boldsymbol{y} = (y_i)$ be their positive Perron vectors such that $A\boldsymbol{x} = \rho\boldsymbol{x}$, $B\boldsymbol{y} = \rho\boldsymbol{y}$. Assume that A and B differ only in the rows having index in the set $\Omega \subset \{1, 2, \ldots, n\}$. Assume that the set Ω and its complement are nonempty. Then*

$$\min_{i\in\Omega} \frac{x_i}{y_i} \leq \frac{x_j}{y_j} \leq \max_{i\in\Omega} \frac{x_i}{y_i}, \quad j = 1, \ldots, n.$$

□

The above result yields information about the variation of the right Perron vector under perturbation of rows. Here we need a sort of dual result concerning the variation of the left Perron vector. The following theorem provides this extension under specific perturbations.

THEOREM 2.2. *Let H be an irreducible adjacency matrix, let (r,s) be a pair of integers such that $h_{r,s} = h_{r,r} = 0$ and q be the number of nonzero entries in the r-th row. Define $\widehat{H} = (\widehat{h}_{i,j})$ such that $\widehat{h}_{r,s} = 1$, $\widehat{h}_{i,j} = h_{i,j}$ otherwise. Let $P = \operatorname{diag}(H\boldsymbol{e})^{-1}H$, $\widehat{P} = \operatorname{diag}(\widehat{H}\boldsymbol{e})^{-1}\widehat{H}$ and denote by $\boldsymbol{p}$ and $\widehat{\boldsymbol{p}}$ their corresponding left Perron vectors. Then*

$$\sigma\frac{\widehat{p}_r}{p_r} \le \frac{\widehat{p}_j}{p_j} \le \frac{\widehat{p}_s}{p_s} \quad j = 1,\dots,n, \tag{2.2}$$

for $\sigma = q/(q+1)$. Moreover,

$$\frac{\widehat{p}_j}{p_j} < \frac{\widehat{p}_s}{p_s}, \quad \text{if } \ h_{r,j} \neq 0 \tag{2.3}$$

and

$$\frac{\widehat{p}_r}{p_r} \le 1 < \frac{\widehat{p}_s}{p_s}. \tag{2.4}$$

Proof. Let D be the diagonal matrix having 1 in the main diagonal except for the r-th diagonal entry which is $\sigma = q/(q+1)$ and observe that

$$\widehat{P} = DP + \frac{1}{q+1}\boldsymbol{e}_r\boldsymbol{e}_s^T.$$

Define $C = D^{-1}\widehat{P}D$ and find that $\boldsymbol{z} = D\widehat{\boldsymbol{p}}$ is a left eigenvector of C, i.e., $\boldsymbol{z}^TC = \boldsymbol{z}^T$, moreover, $z_i = \widehat{p}_i$ for $i \neq r$, $z_r = \sigma\,\widehat{p}_r$. Since

$$C = PD + \frac{1}{q}\boldsymbol{e}_r\boldsymbol{e}_s^T,$$

the matrix C differs from the matrix P only in the columns r and s. Applying Theorem 2.1 with $A = C^T$ and $B = P^T$ yields

$$\min\{\frac{z_r}{p_r}, \frac{z_s}{p_s}\} \le \frac{z_j}{p_j} \le \max\{\frac{z_r}{p_r}, \frac{z_s}{p_s}\}, \quad j = 1,\dots,n,$$

and since $z_r = \sigma\,\widehat{p}_r$, $z_j = \widehat{p}_j$ for $j \neq r$, one gets

$$\min\{\sigma\frac{\widehat{p}_r}{p_r}, \frac{\widehat{p}_s}{p_s}\} \le \frac{\widehat{p}_j}{p_j} \le \max\{\sigma\frac{\widehat{p}_r}{p_r}, \frac{\widehat{p}_s}{p_s}\}, \quad j = 1,\dots,n. \tag{2.5}$$

Now, it is sufficient to prove that $\sigma\widehat{p}_r/p_r < \widehat{p}_s/p_s$ in order to deduce (2.2) from (2.5). Assume by absurd that $\sigma\widehat{p}_r/p_r \ge \widehat{p}_s/p_s$, and deduce from (2.5) that $\widehat{p}_j/p_j \le \sigma\widehat{p}_r/p_r$. Since $p_{r,r} = 0$, and P differs from $\widehat{P}$ only in the rth row, one has $\widehat{p}_r = \sum_i \widehat{p}_{i,r}\widehat{p}_i = \sum_i p_{i,r}\widehat{p}_i$, therefore

$$\widehat{p}_r = \sum_i p_{i,r}p_i\frac{\widehat{p}_i}{p_i} \le \sigma\frac{\widehat{p}_r}{p_r}\sum_i p_{i,r}p_i = \sigma\frac{\widehat{p}_r}{p_r}p_r = \sigma\widehat{p}_r$$

which contradicts the fact that $\sigma < 1$, since $\widehat{p}_r > 0$. The inequality (2.3) follows from the fact that $\sigma < 1$ and $p_{r,j} \neq 0$ since

$$\widehat{p}_j = \sum_i \widehat{p}_{i,j}\widehat{p}_i = \sum_{i\neq r} p_{i,j}\widehat{p}_i + \sigma p_{r,j}\widehat{p}_r < \sum_i p_{i,j}\widehat{p}_i = \sum_i p_{i,j}p_i\frac{\widehat{p}_i}{p_i} \leq p_j\frac{\widehat{p}_s}{p_s}$$

where the last inequality is obtained by replacing $\widehat{p}_i/p_i$ with $\widehat{p}_s/p_s$, in view of (2.2), and using the fact that $\sum_i p_{i,j}p_i = p_j$. Finally, concerning (2.4), if by absurd $\widehat{p}_s/p_s \leq 1$, then (2.2) would imply $\widehat{p}_j/p_j \leq 1$. Since H is irreducible, there exists an integer $j \neq r$ such that $h_{r,j} \neq 0$ that is, in view of (2.3) one obtains $\widehat{p}_j/p_j < 1$. That is, $1 = \sum_j \widehat{p}_j < \sum_j p_j = 1$ which is a contradiction. Similarly we do with the inequality $\widehat{p}_r/p_r \leq 1$. □

The above theorem says that if we introduce a new citation from paper r to paper s, the paper s which receives the citation has an increase of rank greater than or equal to the increase received by any other paper. Moreover, if the paper j is not cited by paper r, i.e., if $h_{r,j} = 0$, then the increase of importance of paper j is strictly less than that of paper s.

Another interesting issue concerns the variation of the Perron vector when a new node is introduced in the graph with a single link to another node. One would expect that the paper that receives the new citation should improve its rank with respect to the other papers. We can provide a formal proof of this fact.

Let V be an $n \times n$ adjacency matrix and denote by $\widetilde{V}$ the $(n+1) \times (n+1)$ matrix having V as leading principal submatrix and zeros in the last row and in the last column. Let H be the $(n+1) \times (n+1)$ matrix having V as leading principal submatrix and having ones in the last row and last column except for the last diagonal entry which is zero. Similarly, define $\widetilde{H}$ the $(n+2) \times (n+2)$ matrix having $\widetilde{V}$ as leading principal submatrix and having ones in the last row and last column except for the last diagonal entry which is zero. Such matrices are depicted below

$$H = \left[\begin{array}{ccc|c} & & & 1 \\ & V & & \vdots \\ & & & 1 \\ \hline 1 & \dots & 1 & 0 \end{array}\right], \quad \widetilde{H} = \left[\begin{array}{ccc|c|c} & & & 0 & \\ & V & & \vdots & 1 \\ & & & 0 & \vdots \\ \cline{4-4} & & & & 1 \\ 0 & \dots & 0 & 0 & \\ \hline 1 & \dots & \dots & 1 & 0 \end{array}\right].$$

Observe that H represents the adjacency matrix of the citation graph associated with V where the dummy paper is added, while $\widetilde{H}$ represents the citation graph associated with the matrix obtained by adding a new paper with no citations, where once again the dummy paper is added after the new insertion.

Both H and $\widetilde{H}$ are irreducible and we can scale their rows to get the stochastic matrices

$$P = \mathrm{diag}(H\boldsymbol{e})^{-1}H, \quad \widetilde{P} = \mathrm{diag}(\widetilde{H}\boldsymbol{e})^{-1}\widetilde{H}.$$

It is a simple matter to show that if $\boldsymbol{x}^T = (x_i)$ is the Perron vector of P, then

$$\widetilde{\boldsymbol{x}} = \theta(x_1, \dots, x_n, \frac{1}{n}x_{n+1}, \frac{n+1}{n}x_{n+1}), \quad \theta = 1/(1 + \frac{2}{n}x_{n+1}),$$

is the Perron vector of $\widetilde{P}$. That is, in their first n components, the two vectors $\boldsymbol{x}$ and $\widetilde{\boldsymbol{x}}$ differ only by the scalar factor θ.

Now, suppose that the new added paper has a citation to paper $s \leq n$. The new adjacency matrix is obtained by setting $\widetilde{h}_{n+1,s} = 1$ in the matrix $\widetilde{H}$. Let us denote by $\widehat{H}$ the matrix obtained in this way and by $\widehat{P} = \mathrm{diag}(\widehat{H}\boldsymbol{e})^{-1}\widehat{H}$ the stochastic matrix obtained by scaling the rows of $\widehat{H}$. Applying Theorem 2.2 to $\widetilde{H}$ and to $\widehat{H}$ yields the following

THEOREM 2.3. *For the Perron vectors $\boldsymbol{x}$ and $\widetilde{\boldsymbol{x}}$ of the matrices P and $\widetilde{P}$, respectively, it holds*

$$\sigma\frac{\widehat{x}_{n+1}}{x_{n+1}} \leq \frac{\widehat{x}_j}{x_j} < \frac{\widehat{x}_s}{x_s}, \quad j = 1, \ldots, n, \quad j \neq s,$$

for $\sigma = 2/3$. Moreover, $\widehat{x}_s/x_s > 1 + \frac{2}{n}x_{n+1}$.

Proof. From Theorem 2.2 applied to $\widetilde{P}$ and $\widehat{P}$, with $r = n+1$ and $q = 2$, one obtains $\sigma\frac{\widehat{x}_{n+1}}{\widetilde{x}_{n+1}} \leq \frac{\widehat{x}_j}{x_j} < \frac{\widehat{x}_s}{\widetilde{x}_s}, \quad j = 1, \ldots, n, \quad j \neq s$ where the second inequality is strict since $p_{n+1,j} = 0$ for $j \neq r, n+2$. The theorem holds since $\widetilde{x}_i = x_i/(1+\frac{2}{n}x_{n+1})$. □

The above theorem says that if we introduce a new paper which contains a citation to paper s, then paper s has an increase of importance which is strictly greater than the increase of importance reached by the other papers. Perturbation analysis of the Perron vector for a stochastic irreducible matrix has been addressed in [12] whit a specific attention to PageRank. However, our results have a different flavor since we are interested the rank index of the subjects rather then the values of the eigenvector's entries.

In the case we have a new paper that cites d papers from Theorem 2.1 we have that *at least* one of the cited papers will have an increase of importance greater than that of the other papers. However we cannot say that all the cited papers will increase their rank more than the non-cited ones.

3. Two-class model. Consider the case where besides papers we wish to rank authors. We can make this in an integrated model where a paper, besides giving importance to the papers that it cites, gives importance to its authors, and where an author gives importance to the papers that he/she has written and to his/her co-authors. This approach is similar to Kleinberg's idea [11] of Hub and Authorities for ranking Web pages, which can be reformulated in terms of a symmetric block matrix as described in [3].

As in the one-class model, we require that the amount of importance given by each subject to all the others is equal to the importance of the subject itself. That is, the importance is neither destroyed nor created. This corresponds to deal with nonnegative matrices which are row-stochastic.

Assume we have m authors numbered from 1 to m. Besides the adjacency matrix H concerning paper citations, we introduce the $m \times n$ matrix $K = (k_{i,j})$, concerning authorship, such that $k_{i,j} = 1$ if the author i is (co)author of the paper j, $k_{i,j} = 0$ otherwise. Define the matrix $A = KK^T = (a_{i,j})$. By a simple inspection, it turns out that $a_{i,j}$ is the number of papers which are co-authored by authors i and j.

Observe that by definition, any author has at least a paper so that the matrix K cannot have null rows and it can be made row-stochastic. As in the case of the one-class model, the matrix H might have null rows, therefore we proceed as we did in Section 2 by introducing a dummy paper with the same features as before. In addition, we assume that this paper is co-authored by all the existing authors. The

introduction of this new paper favors neither any specific author nor any specific paper.

Now, let us still denote with n the number of all the papers, included the dummy paper, and introduce the $(m+n)\times(m+n)$ matrix S which collects the information about citation and co-authorship

$$S = \begin{bmatrix} KK^T & K \\ K^T & H \end{bmatrix}$$

where H is the $n \times n$ adjacency matrix of papers introduced in Section 2, while the $m \times n$ matrix K contains the information about the co-authorship. Recall that, due to the dummy paper, the last column of K is made by all ones, i.e., all the authors are coauthors of the dummy paper, moreover, the matrix S is irreducible.

The role of K^T in the lower leftmost block of S is that, for $i > m$ and $j \le m$, $s_{i,j}$ is an entry of K^T and this entry is nonzero if and only if the corresponding paper $i-m$ has the author j as (co)author. In other words, the matrix S captures the relationships of authorship and citation among the different subjects (authors and papers) of this model, so that $s_{i,j} = 0$ if there exists no relationship between subject i and subject j. The kind of relationship, i.e., either citation or authorship, is determined by the kind of classes the subjects i and j belong to.

Example 2. Consider Example 1 where four different authors are added with the following authorship: author 1 has written papers 1 and 4; author 2 has written papers 2 and 4; author 3 has written papers 3 and 4; author 4 has written papers 5 and 6. In this way, the matrix K is given by

$$K = \left[\begin{array}{cccccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right]$$

including the dummy paper, and the full matrix S is

$$S = \left[\begin{array}{cccc|ccccccc} 3 & 2 & 2 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 3 & 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 2 & 2 & 3 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 3 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ \hline 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

. □

The matrix S is a sort of generalized adjacency matrix where each block $A = KK^T$, K, K^T and H cannot have null rows. It is a simple matter to scale by rows S to obtain a stochastic matrix to be used as weight matrix for distributing the importance from a subject to another by means of citation or authorship. However, due to the different nature of the two classes *Authors* and *Papers*, it is more convenient

to make stochastic each one of the four blocks and to use suitable parameters to tune the influence of authorship over the influence of citations.

Acting as we did in Section 2 we scale the rows of the four blocks in such a way to obtain four stochastic matrices and use the coefficients of such matrices to weight the amount of importance that each subject belonging to either the class *Papers* or to the class *Authors* provides to the other subjects. More precisely, define

$$Q = \begin{bmatrix} \operatorname{diag}(A\boldsymbol{e})^{-1}A & \operatorname{diag}(K\boldsymbol{e})^{-1}K \\ \operatorname{diag}(K^T\boldsymbol{e})^{-1}K^T & \operatorname{diag}(H\boldsymbol{e})^{-1}H \end{bmatrix},$$

where $A = KK^T$ and the symbol $\boldsymbol{e}$ denotes the vector of all ones of dimension m, n or $m+n$ depending on the context, let $\Gamma = (\gamma_{i,j})$ be a 2×2 row-stochastic matrix, and consider the matrix

$$P = Q \odot \Gamma = \begin{bmatrix} \gamma_{1,1}\ \operatorname{diag}(A\boldsymbol{e})^{-1}A & \gamma_{1,2}\ \operatorname{diag}(K\boldsymbol{e})^{-1}K \\ \gamma_{2,1}\ \operatorname{diag}(K^T\boldsymbol{e})^{-1}K^T & \gamma_{2,2}\ \operatorname{diag}(H\boldsymbol{e})^{-1}H \end{bmatrix}. \tag{3.1}$$

For the sake of notational simplicity, given a $q \times q$ block matrix $A = (A_{i,j})$ and a $q \times q$ matrix $B = (b_{i,j})$ we denote by $A \odot B$ the $q \times q$ block matrix having blocks $b_{i,j}A_{i,j}$.

We have the following

PROPOSITION 3.1. *For $A = (A_{i,j})_{i,j=1,n}$, where $A_{i,j}$, $i, j = 1, n$ are row stochastic, and for a row stochastic matrix $B = (b_{i,j})_{i,j=1,n}$ it holds that $A \odot B$ is row stochastic.*

Proof. One has

$$A \odot B\boldsymbol{e} = \begin{bmatrix} \sum_i b_{1,i}A_{1,i}\boldsymbol{e} \\ \vdots \\ \sum_i b_{n,1}A_{n,i}\boldsymbol{e} \end{bmatrix} = \begin{bmatrix} \sum_i b_{1,i}\boldsymbol{e} \\ \vdots \\ \sum_i b_{n,i}\boldsymbol{e} \end{bmatrix} = \boldsymbol{e}$$

□

In particular, the matrix P in (3.1) is row stochastic. In this way we can define our model by means of the eigenvalue equation

$$\boldsymbol{p}^T = \boldsymbol{p}^T P \tag{3.2}$$

with P being the matrix in (3.1), where in the vector $\boldsymbol{p}$ the first m components describe the importance of the authors while the remaining components describe the importance of the papers.

Equation (3.2) states that the importance of a paper is the sum of the importances given by the authors of the paper, weighted with the factor $\gamma_{1,2}$ plus the importance given by the citations received by the paper, weighted with the factor $\gamma_{2,2}$. Similarly, the importance of an author is the sum of the importances received by the coauthors, weighted with the factor $\gamma_{1,1}$ plus the importances received by the papers that he/she has written, weighted by the factor $\gamma_{2,1}$. More precisely, in components one has:

$$p_j = \gamma_{1,1} \textstyle\sum_{i=1}^m p_i \dfrac{a_{i,j}}{\sum_t a_{i,t}} + \gamma_{2,1} \displaystyle\sum_{i=1}^n p_{m+i} \frac{k_{j,i}}{\sum_t k_{t,i}} \qquad j = 1, \ldots, m \qquad \text{authors}$$

$$p_j = \gamma_{1,2} \textstyle\sum_{i=1}^m p_i \dfrac{k_{i,j}}{\sum_t ki,t} + \gamma_{2,2} \displaystyle\sum_{i=1}^n p_{m+i} \frac{h_{i,j}}{\sum_t h_{i,t}} \qquad j = m+1, \ldots, n \qquad \text{papers}$$

If the matrix Γ is irreducible, then also P is irreducible and the vector $\boldsymbol{p}$ normalized such that $\sum p_i = 1$ exists and is unique. Observe also that since it is meaningless to compare subjects of different classes, namely authors and papers, the normalization of $\boldsymbol{p}$ is still meaningful if restricted separately to the subvector containing the first m components and the subvector containing the remaining n components.

It is interesting to point out that, denoting $\mu_A = \sum_{i=1}^m p_i$ and $\mu_P = \sum_{i=1}^n p_{m+i}$ the overall amount of the importance of authors and of papers, respectively, it holds that the vector (μ_A, μ_P) is a left eigenvector of Γ corresponding to the eigenvalue 1. Moreover, if we replace the matrix Γ with $\Gamma' = D\Gamma D^{-1}$, where D is any nonsingular diagonal matrix, then the left Perron vector $\boldsymbol{p}'$ of $P' = Q \odot \Gamma'$ differs from $\boldsymbol{p}$ only for the values of μ_A and μ_P. Therefore, in order to evaluate separately the subvectors of $\boldsymbol{p}$ related to authors and papers, respectively, it is enough to consider a matrix Γ of the kind

$$\begin{bmatrix} 1-\alpha & \alpha/\theta \\ \beta\theta & 1-\beta \end{bmatrix}$$

for α and β suitable scalars in $[0,1]$ and $\theta > 0$ any arbitrary constant. In particular, we may chose $\theta = \alpha/\beta$ which makes Γ column-stochastic, or $\alpha = \sqrt{\alpha/\beta}$ which makes Γ symmetric.

The parameters $\gamma_{i,j}$ determine the amount of influence that each class has on the other classes. In particular, choosing $\Gamma = I$ provides an uncoupled problem where the matrix P is block diagonal. In this case, the ranking of papers is independent of that of authors and coincides with the ranking obtained in the one-class model of Section 2. In this special case, the authors receive importance only from authorship and not from the importance of their papers.

We observe that this model has an annoying drawback. Namely, the importance received by a paper from its coauthors is proportional to the number of coauthors. The larger is this number, the larger is the amount of importance received through the authorship. In this way, a paper having many authors might result to be more important than a paper having a single author even though the former has much less citations. This drawback, which is well illustrated in the next example, can be easily removed by normalizing the block $(1,2)$ of P by columns. This corresponds to evaluate the importance received by the co-authorship as the *mean*, instead of the sum, of the importances brought to the paper by the coauthors.

Observe that the block $\widehat{K} = K\,\mathrm{diag}(K^T\boldsymbol{e})^{-1}$ that we would obtain by normalizing the matrix K this way is not stochastic anymore. A way to obtain a row-stochastic block $\widetilde{K}$ after the normalization by columns is to use the following:

Algorithm 1. For each $i \in \{1,\dots,m\}$, compute $s_i = \sum_{j=1}^{n-1} \widehat{k}_{i,j}$.

If $s_i \le \widehat{k}_{i,n} = 1 - 1/m$, set $\widetilde{k}_{i,j} = \widehat{k}_{i,j}$, for $j = 1,\dots,n-1$, and $\widetilde{k}_{i,n} = 1 - s_i$.

Else, divide the ith row of $\widetilde{K}$ by the sum of its entries, that is, set

$$\widetilde{k}_{i,j} = \widehat{k}_{i,j}/(s_i + 1/m).$$

Output $\widetilde{K} = (\widetilde{k}_{i,j})$.

It is immediate to verify that for $s_i = 1 - 1/m$ the two different normalizations described in the above algorithm provide the same result. Observe also that the normalization for $s_i \le \widehat{k}_{i,n} = 1 - 1/m$, leaves unchanged the amounts of importance that author i yields to papers j, $j = 1,\dots,n-1$ after the column scaling, and assigns

to the dummy paper the remaining amount of importance that is missing, that is, $1 - s_i$.

One could think that the column normalization followed by a row normalization is enough to get a row-stochastic matrix where the importance that a paper receives by its authors is the average of the importances of the authors. Unfortunately this is false in general. Consider for instance a matrix K where the first column has the first q entries equal to 1 and the remaining zero and where the first q rows have zeros except for their first and last entry. The column normalization would transform the ones in the first column into $1/q$ and the ones in the last column into $1/m$. But the subsequent row normalization would turn the entries in the first column into $m/(q+m)$ and the ones in the last column into $q/(m+q)$. For instance, if $q = m/2$ then each one of the first q authors would give $2/3$ of its importance to the first paper instead of $1/q$ as we desired.

The matrix that we obtain by row normalization and by applying Algorithm 1 is the following

$$P = \begin{bmatrix} \gamma_{1,1}\ \mathrm{diag}(A\boldsymbol{e})^{-1}A & \gamma_{1,2}\ \widetilde{K} \\ \gamma_{2,1}\ \mathrm{diag}(K^T\boldsymbol{e})^{-1}K^T & \gamma_{2,2}\ \mathrm{diag}(H\boldsymbol{e})^{-1}H \end{bmatrix} \tag{3.3}$$

where $\widetilde{K}$ is the matrix obtained by means of Algorithm 1

Example 3. In order to understand the differences of the two models, consisting in the way one normalizes block (1,2), let us consider the case of Example 2 with weights $\gamma_{i,j} = 1/2$, $i, j = 1, 2$. Computing the Perron vector in the model described in (3.1) we have that the first 4 components of the left Perron vector of the matrix P (the ones corresponding to authors) are given by

$$(0.238912,\ 0.238912,\ 0.238912,\ 0.283265)$$

where we have normalized to sum 1. The remaining 7 components (the ones corresponding to papers) are

$$(0.0778083,\ 0.0778083,\ 0.0778083,\ 0.176898,\ 0.104652,\ 0.145862,\ 0.339163)$$

where we have normalized again to sum 1. Observe that the first three authors have the same rank while the fourth author has a higher rank. In fact, he/she is the author of two important papers and receives the importance of this latter. Moreover, the first three papers still keep the same rank as in the one-class model, but the fourth and the fifth papers have different ranks, in particular, the fourth paper reaches the maximum rank followed by paper 6 and 5. The reason is that paper 4 has many authors which sum up their importances.

By following the model described in (3.3), where the average of the importances of the authors is considered in place of their sum, one obtains

$$(0.237763, 0.237763, 0.237763, 0.28671)$$

for authors and

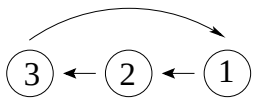
$$(0.11009, 0.11009, 0.11009, 0.137613, 0.126243, 0.150923, 0.25495)$$

for papers. This time, as one would have expected, paper 6 is the one with the highest rank while paper 4 is more important than paper 5. The fourth author has still a

rank higher than those of the remaining authors. □

The following example shows that on a basis of equivalent papers, an author with more papers has a higher importance.

EXAMPLE 4. Consider the simple case of three papers with a cyclic graph of citations as shown below.



All of them have a single citation and the adjacency citation matrix H is given by

$$H = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ \hline 1 & 1 & 1 & 0 \end{array}\right]$$

including the dummy paper In the one-class model the three papers have the same importance. In fact, the computed vector p, including the dummy component, is given by

$$\boldsymbol{p}^T = (0.222222,\ 0.22222,\ 0.222222,\ 0.333333).$$

In the two-class model, assuming that there are 3 authors and that each paper has a single different author, the matrix K is given by

$$K = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

and the matrix $A = KK^T$ is

$$KK^T = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

The computed Perron vector with weights $\gamma_{i,j} = 1/2$ is

$$(0.333333,\ 0.333333,\ 0.333333)$$

for authors and

$$(0.233333,\ 0.233333,\ 0.233333,\ 0.3)$$

for papers. We can see that all the papers, except for the dummy, as well all the authors have the same rank.

Now assume that there are three authors; author i is author of paper i for $i = 1, 2, 3$, moreover, author 1 is also coauthor of paper 3. In the two-class model, author 1 is expected to receive more importance than the other authors since he/she has written more papers of roughly the same rank. This implies that also his/her two papers should slightly increase their importance. With this data the matrices K and A are given by

$$K = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad A = KK^T = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}.$$

In fact, with $\gamma_{i,j} = 1/2$, the computed vector $\boldsymbol{p}$ in the part concerning authors is

$$(0.423170,\ 0.302289,\ 0.274541)$$

and in the part concerning papers, dummy paper included, is

$$(0.226729,\ 0.222693,\ 0.234666,\ 0.315913).$$

Author 1 has increased his/her importance together with the papers coauthored by him/her. This confirms the consistency of our model. □

Example 5. Consider the same case of Example 1 and assume that author i is (co)author of paper i for $i = 1, 2, 3, 4, 5, 6$, while author 6 is coauthor of paper 1. From the graph of citations we expect that paper 6 is the most important (and this is true in the one-class model), we expect that author 6 has higher rank and that he/she lifts up the rank of paper 1 of which is coauthor. In this case the matrices K and $A = KK^T$ are given by

$$K = \left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right], \quad KK^T = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 1 \\ 2 & 1 & 1 & 1 & 1 & 3 \end{bmatrix}.$$

A numerical computation performed with $\gamma_{i,j} = 1/2$, shows that the vector $\boldsymbol{p}$ in the components concerning authors is

$$(0.136510,\ 0.150226,\ 0.149926,\ 0.167705,\ 0.167705,\ 0.227929)$$

and in the components concerning papers is

$$(0.103592,\ 0.100157,\ 0.099599,\ 0.132617,\ 0.132617,\ 0.157513,\ 0.273905).$$

Once again the result of the computation confirms the consistency of the model. □

It is possible to show that Theorems 2.2 and 2.3 still hold for the matrix P defined by (3.3) if the perturbation concerns an entry in the lower rightmost block of P.

4. Three-class model. Besides the classes of *Papers* and *Authors*, now we introduce the class of *Journals* of cardinality q and we number the elements of this set from 1 to q. Together with the matrices H and K, we consider the matrix $F = (f_{i,j})$ such that $f_{i,j} = 1$ if journal i publishes paper j and $f_{i,j} = 0$ otherwise, and the matrix $G = (g_{i,j})$ such that $g_{i,j} = r$ if the author j has published r papers in the journal i. Similarly define $E = (e_{i,j})$ the matrix such that $e_{i,j}$ is the number of citations from papers published in journal i to papers published in journal j. A direct inspection shows that

$$E = FHF^T, \quad G = FK^T.$$

The full adjacency matrix which collects all the information about citation, authorship and publications is given by

$$S = \begin{bmatrix} E & G & F \\ G^T & A & K \\ F^T & K^T & H \end{bmatrix} \tag{4.1}$$

Similarly to the two-case model, S synthesizes the relationship between the different subjects of our model (journals, authors and papers) in such a way that $s_{i,j} \neq 0$ if there exists a relationship between subject i and subject j. The kind of relationship depends on the pair of classes which the subjects i and j belong to.

Also in this case we normalize each block of S by scaling its rows in such a way to obtain stochastic matrices and use a 3×3 stochastic matrix of parameters to better tune the influence of a class on the other ones.

In order to make this, we need that each block does not have an entire row of zeros. This possibility was avoided in the previous models by introducing a dummy paper. Here, we can do the same. Since we have to avoid creating privileges among the subjects, we may proceed in two different ways. Either we assume that the dummy paper is published by all the journals, or that there exists a dummy journal which publishes only the dummy paper.

With these two choices we get two different models represented by two suitable modifications of the matrix S of (4.1), where the rank vector is given by the Perron vector of a suitable stochastic matrix. The analysis of these models is left to a future work.

5. Numerical tests. We tested the approaches discussed in previous sections using the CiteSeer dataset which can be freely downloaded from the CiteSeer web site. CiteSeer is a scientific literature digital library and search engine that focuses primarily on the literature in computer and information science [9].

CiteSeer crawls and gathers academic and scientific documents on the web and uses autonomous citation indexing to permit querying by citation or by document and then ranking them by citation impact.

For our experiments we use the CiteSeer index downloaded on June 2007 consisting of about 800,000 papers. This dataset was first cleaned to remove some incorrect references, such as items without an author or isolated items. We obtained a dataset consisting of approximately 250,000 authors and 350,000 papers in XML format. The data have been then parsed producing the matrix H and the matrix K.

Despite every item in the XML format contains much information, it is not easy to recover the journal where the paper has been published because the journals have not associated a unique identifier. This means that with these data we were not able to test the effectiveness of our three-class model. However, experimental results on the MR [1] dataset [1], prove that also the ranking of journals is very good, and able to capture concepts such as prestige and authoritativeness.

We present the results of two different numerical tests. The first test addresses the problem of the ranking of papers by using the one-class model. The results, reported in Table 5.1 shows the top 6 papers obtained with our model. We can recognize among these papers great pieces of work such as fundamental papers in cryptography, the paper by Bryant introducing the binary decision diagram (BDD), a data structure for describing boolean functions, or the paper in which the TCP/IP protocol has been proposed.

We see that the position occupied in our ranking by these papers, doesn't coincide with that occupied by simply sorting the papers for descending number of citations received. This is due to the fact that here, not all the citations are regarded as the same, but citation by important papers have a greater weight. For example it is

[1] The AMS didn't give us the authorization to publish results obtained with part of their index.

paper	pos.	cit.
Diffie, Hellman- New directions in Cryptography	31	553
Rivest, Shamir, Adleman - Public Key cryptography	3	1218
Bryant -Boolean Functions Manipulation, BDD	1	1636
Kirkpatrick, Gelatt, Vecchi- Simulated Annealing	2	1337
Floyd, Jacobson - TCP/IP Protocol	4	1125
Canny - Computational approach to Edge detection	10	834

TABLE 5.1

Experimental Results for the one-class model. In the first column papers in order of decreasing rank are listed with the name of the authors and a short identification of the title of the paper. The second column contains the position in the list ordered by decreasing number of citations, and third column the number of citation received by the paper.

Author	num. cit	num. pap.	av. num. cit.
Randal Bryant	2615	83	31.5
Sally Floyd	4950	91	54.4
John K. Ousterhout	2214	23	96.3
Luca Cardelli	2112	91	23.2
Van Jacobson	4719	40	118.0
Rakesh Agrawal	4745	83	57.2
Jack J. Dongarra	2799	291	9.6
Raj Jain	1038	116	8.9
Douglas C. Schmidt	2980	329	9.1
Vern Paxson	2735	66	41.4
John Mccarthy	911	41	22.2
Thomas A. Henzinger	3694	176	21.0

TABLE 5.2

Experimental Results for the two-class model for the Subject Author. In the first column the top authors are ranked in decreasing order of rank. In the remaining columns are reported the number of citation received, the number of papers by the author and indexed in the dataset, and the average number of citation per paper.

possible to see that the paper by Diffie, and Hellman is contained in the reference list of the paper by Rivest, Shamir and Adleman, and hence it gets an higher rank even if it receives less citations.

In Table 5.2 we reporte the top authors obtained by choosing uniform weights. We can recognize very important computer scientists who wrote important papers in many areas of the information sciences. Some of the authors in the list rank higher than one would expect, mainly because they have important co-authors. However, we can smooth the effect of co-authorship by reducing the corresponding coefficient in the weight matrix.

In Table 5.3 we reporte the results for the subject *Paper* obtained with uniform weights. The differences with Table 5.1 are essentially in the order of the best papers, that in the two-class model are influenced also by the authoritativeness of the authors.

6. Conclusions and open problems. In this paper we proposed integrated models for evaluating papers, authors and journals based on citations, coauthorship

paper	pos.	cit.
Kirkpatrick, Gelatt, Vecchi- Simulated Annealing	2	1337
Bryant - Boolean Functions Manipulation, BDD	1	1636
Rivest, Shamir, Adleman - Public key cryptography	3	1218
Canny - Computational approach to Edge detection	10	834
Floyd, Jacobson - TCP/IP Protocol	4	1125
Diffie, Hellman- New directions in Cryptography	31	553
John K. Ousterhout - Tcl and the Tk Toolkit	8	913
Harel - Statecharts formalism	6	1042
Elman - Neural Networks	26	589
Jones - Vienna Development Method	23	609

TABLE 5.3

Experimental Results for the two-class model for the Subject Paper. In the first column papers in order of decreasing rank are listed with the name of the authors and a short identification of the title of the paper. The second column contains the position in the list ordered by decreasing number of citations, and third column the number of citation received by the paper.

and publications. After the one-class model for ranking scientific publications, we have introduced the two-class model which ranks papers and authors, and the three-class model for ranking papers, authors and journals. In all the models, the rank vector is obtained as the Perron vector of an irreducible stochastic matrix.

Some theoretical results have been proved concerning the variation of the Perron vector of an irreducible stochastic matrix under limited changes of its entries. These results prove that the model behaves as one would expect when a new citation occurs.

Simple examples show that our model is more suited for ranking scientific publications than the known models based only on the number of citations.

Some open issues remain to be analyzed. A theoretical issue concern perturbation theorems. In fact, in Section 2 we proved that if a paper receives a new citation then its rank increases more than the rank of the other papers do. It would be natural to guess that if more than one paper receives a citation, then *all* the cited papers increase their importance more than the other papers do. At the moment a proof of this property is missing and no counterexample is known. We plan to address this problem in our future work.

A second issue which deserves attention is reletated to the "static" nature of our model. That is, the time of publication of the papers or the time a citation is received do not play any role. It is commonly accepted that a recent paper and an old paper which receive the same number of citations should not have the same rank. In fact, the citations received by the old paper are relatively less important than the citations received by the recent paper. We are currently investigating this issue trying to insert the factor "time" in our model.

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