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Abstract

Well-known metric spaces for modelling finitely branching and image finite systems are shown to be (the carrier of) terminal coalgebras.

Introduction

In the area of *metric semantics*, various metric structures have been proposed to model a wide spectrum of programming notions (see, e.g., [BV96]). In this paper, we focus on metric structures for modelling *nondeterministic* systems which may give rise to both *terminating* and *nonterminating* computations. The systems we have in mind are *labelled transition systems* [Kel76]. A large variety of programming notions can be modelled by means of these systems (see, e.g., [Plo81]). The models we consider are *linear* (cf. [Pnu85]). In these models, the locations in a computation where a nondeterministic choice is made are not visible. These linear models are usually contrasted with *branching* models (cf. [Gla90]). In those models, the positions in the computation where a nondeterministic choice is made are administrated.

Typical examples of linear metric structures proposed in the literature are sets of *words* (see, e.g., [Niv79]) and sets of *pomsets* (see, e.g., [BW90]). Other examples can be found in, e.g., [BW91]. Here, we concentrate on sets of finite and infinite words. The words over a set A of actions, denoted by A^∞, are provided with a Baire-like metric [Bai09]. The distance between two words is given in terms of the length of their longest common prefix. The set $\mathcal{P}_n(A^\infty)$ of nonempty sets of words is endowed with the induced Hausdorff metric [Hau14]. This space is not a metric space, but only a pseudometric space. The restriction to the subspaces $\mathcal{P}_{nk}(A^\infty)$ of nonempty and *compact* sets of words and $\mathcal{P}_{nc}(A^\infty)$ of nonempty and *closed* sets of words gives us a *complete metric space* [Kur56, Hah32].

Like in automata theory, one can associate to a labelled transition system $\langle S, A, \rightarrow, \downarrow\rangle$—where S is the (possibly infinite) set of states, A is the (possibly infinite) set of actions, $\rightarrow$ is the transition relation, and $\downarrow$ tells us in which states a computation may (but not necessarily has to) terminate—and an (initial) state $s \in S$, the corresponding language

$$\{\, a_1 a_2 \ldots a_n \mid s = s_0 \xrightarrow{a_1} s_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} s_n \downarrow \,\} \cup \{\, a_1 a_2 \ldots \mid s = s_0 \xrightarrow{a_1} s_1 \xrightarrow{a_2} \cdots \,\}.$$

In this way we assign to each system and state of the system a point of the linear space $\mathcal{P}_n(A^\infty)$. These points we call the *linear processes*. The subspace $\mathcal{P}_{nk}(A^\infty)$ is well-suited for handling *finitely branching* labelled transition systems—a system is finitely branching if every state has only finitely many outgoing transitions—and the subspace $\mathcal{P}_{nc}(A^\infty)$ is used to deal with *image finite* labelled transition systems—a system is image finite if every state has only finitely many outgoing transitions labelled by the same action. Reminiscent to the classical result linking finite automata and regular languages [Kle56], finitely branching systems correspond to the points of the space $\mathcal{P}_{nk}(A^\infty)$—therefore we call these points the *finitely branching* linear processes—and image finite systems correspond to the points of the space $\mathcal{P}_{nc}(A^\infty)$—the points of this space are called the *image finite* linear processes. These results are folklore (see, e.g., [Lan69]) and are based on König's lemma [Kön26].

During the last decade the insight gradually grew that systems like the above mentioned labelled transition systems can be described as *coalgebras*. Among these coalgebras (of an endofunctor on a category), the *terminal* one plays an important role. It provides us with *definitions* and *proofs* by *coinduction* (see, e.g., [JR97]). The branching metric structures introduced in [BZ82, BZ83] were already known to be the carrier of terminal coalgebras (see [RT92], cf. [Acz88, Bar93]). Here we show that also the above mentioned linear metric structures are. This result can be exploited by coinductively defining operations on the metric spaces (e.g., the merge) and by coinductively proving properties of these operations (e.g., the commutativity of the merge). We do not provide the reader with such an example, because the examples presented in, e.g., [JR97] can be adapted to our setting straightforwardly. Our observation that the metric spaces of linear processes are terminal coalgebras shows that these spaces fit into the general coalgebra framework.

Related linear structures have been studied in, e.g., [HP79, TJ93, RT93] in an order- and set-theoretic setting. In those papers, only finitely branching linear processes are considered. Here we also deal with image finite ones. In the other papers, the structures involved are supplied with a join operation. Also the metric spaces $\mathcal{P}_{nk}(A^\infty)$ and $\mathcal{P}_{nc}(A^\infty)$ have a natural join: the set-theoretic union. Whether all this can also be carried out in a setting where the (complete) metric spaces are supplied with a (nonexpansive) join operation and how this relates to the work presented here is left for future research.

The rest of this paper is organized as follows. In Section 1, we introduce the metric spaces $\mathcal{P}_{nk}(A^\infty)$ and $\mathcal{P}_{nc}(A^\infty)$. These metric spaces are shown to be the carrier of terminal coalgebras in Section 2. The reader is assumed to have some basic knowledge of metric spaces and category theory.

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1 The metric spaces $\mathcal{P}_{nk}(A^\infty)$ and $\mathcal{P}_{nc}(A^\infty)$

The well-known complete metric spaces $\mathcal{P}_{nk}(A^\infty)$ and $\mathcal{P}_{nc}(A^\infty)$ of finitely branching and image finite linear processes are introduced. Furthermore, some simple operations on complete metric spaces[1], which we need to define the functors in the next section, are presented.

To define the spaces $\mathcal{P}_{nk}(A^\infty)$ and $\mathcal{P}_{nc}(A^\infty)$ we first endow the set A^∞ of finite and infinite words over the nonempty set A of actions with the following Baire-like metric [Bai09].

Definition 1 The function $d_{A^\infty} : A^\infty \times A^\infty \to [0,1]$ is defined by

$$d_{A^\infty}(w_1, w_2) = \begin{cases} 0 & \text{if } w_1 = w_2 \\ 2^{-n} & \text{otherwise,} \end{cases}$$

where n is the length of the longest common prefix of w_1 and w_2. ⌟

Exercise 2 Check that A^∞ is a complete metric space. ⌟

Next, we endow the set $\mathcal{P}_n(A^\infty)$ of nonempty sets of words with the induced Hausdorff metric [Hau14]. This only gives us a pseudometric space but not a metric space. By restricting ourselves to the subspaces $\mathcal{P}_{nk}(A^\infty)$ of nonempty and compact sets of words and $\mathcal{P}_{nc}(A^\infty)$ of nonempty and closed sets of words we do get a metric space. On these subspaces the induced Hausdorff metric amounts to the following.

[1] For the metric spaces $\langle X, d_X\rangle$ we encounter in this paper, the set X is assumed to be nonempty and the metric d_X is presupposed to be 1-bounded. To simplify notations, we shall sometimes write X instead of $\langle X, d_X\rangle$.

DEFINITION 3 The function $d_{\mathcal{P}_{nk}(A^\infty)} : \mathcal{P}_{nk}(A^\infty) \times \mathcal{P}_{nk}(A^\infty) \to [0,1]$ is defined by

$$d_{\mathcal{P}_{nk}(A^\infty)}(W_1, W_2) = \max\{\max_{w_1 \in W_1} \min_{w_2 \in W_2} d_{A^\infty}(w_1, w_2), \max_{w_2 \in W_2} \min_{w_1 \in W_1} d_{A^\infty}(w_2, w_1)\}$$

and the function $d_{\mathcal{P}_{nc}(A^\infty)} : \mathcal{P}_{nc}(A^\infty) \times \mathcal{P}_{nc}(A^\infty) \to [0,1]$ is defined by

$$d_{\mathcal{P}_{nc}(A^\infty)}(W_1, W_2) = \max\{\sup_{w_1 \in W_1} \inf_{w_2 \in W_2} d_{A^\infty}(w_1, w_2), \sup_{w_2 \in W_2} \inf_{w_1 \in W_1} d_{A^\infty}(w_2, w_1)\}.$$

$\lrcorner$

Note that in the compact case, we can replace sup and inf by max and min, respectively.

PROPOSITION 4 (KURATOWSKI AND HAHN) *$\mathcal{P}_{nk}(A^\infty)$ and $\mathcal{P}_{nc}(A^\infty)$ are complete metric spaces.*

PROOF See [Kur56, Lemma 3] and [Hah32, § 9.6 and § 18.10]. □

We conclude this section with some simple operations on complete metric spaces. We start with an elementary

EXAMPLE 5 The set $1 = \{0\}$ with the obvious metric d_1 is a complete metric space. $\lrcorner$

The operation that leaves the set unchanged and multiplies the metric by a $\frac{1}{2}$ is considered in

EXERCISE 6 Let $\langle X, d_X \rangle$ be a complete metric space. Verify that $\langle X, \frac{1}{2} d_X \rangle$ is also a complete metric space. $\lrcorner$

Given a nonempty set I and a complete metric space $\langle X, d_X \rangle$, we turn the set $I \to X$ of functions from I to X into a complete metric space as follows.

DEFINITION 7 The function $d_{I \to X} : (I \to X) \times (I \to X) \to [0,1]$ is defined by

$$d_{I \to X}(f_1, f_2) = \sup_{i \in I} d_X(f_1(i), f_2(i)).$$

$\lrcorner$

EXERCISE 8 Check that $I \to X$ is a complete metric space. $\lrcorner$

Let I be a nonempty set and, for all $i \in I$, let $\langle X_i, d_{X_i} \rangle$ be a complete metric space. By $\coprod_{i \in I} X_i$ we denote the disjoint union of the X_i's. The elements of this disjoint union are written as $\langle i, x \rangle$ where $x \in X_i$ for $i \in I$. Instead of $\coprod_{i \in \{0,1\}} X_i$ we usually write $X_0 \amalg X_1$ and we sometimes use $2 \cdot X$ to denote $X \amalg X$.

DEFINITION 9 The function $d_{\coprod_{i \in I} X_i} : \coprod_{i \in I} X_i \times \coprod_{i \in I} X_i \to [0,1]$ is defined by

$$d_{\coprod_{i \in I} X_i}(x_1, x_2) = \begin{cases} d_{X_i}(x_1, x_2) & \text{if } x_1, x_2 \in X_i \\ 1 & \text{otherwise.} \end{cases}$$

$\lrcorner$

EXERCISE 10 Prove that $\coprod_{i \in I} X_i$ is a complete metric space. $\lrcorner$

2 $\mathcal{P}_{nk}(A^{\infty})$ and $\mathcal{P}_{nc}(A^{\infty})$ are terminal coalgebras

A category *CMS* of complete metric spaces and endofunctors $\mathcal{FB}$ and $\mathcal{IF}$ on this category are introduced. Both functors are shown to have a unique (up to isomorphism) fixed point which is a terminal coalgebra. Furthermore, the space $\mathcal{P}_{nk}(A^{\infty})$ of finitely branching linear processes and the space $\mathcal{P}_{nc}(A^{\infty})$ of image finite linear processes are proved to be fixed points of $\mathcal{FB}$ and $\mathcal{IF}$, respectively.

DEFINITION 11 The category *CMS* has complete metric spaces as objects and nonexpansive functions as arrows. $\lrcorner$

EXERCISE 12 Verify that *CMS* is indeed a category. Prove that $\coprod_{i \in I} X_i$ is a coproduct object in *CMS*. $\lrcorner$

Obviously, $\coprod$ can be extended to a functor. Also the constant 1 can be turned straightforwardly into a functor. The extension of the operations $\frac{1}{2}$ and $I \rightarrow$ to functors is left as

EXERCISE 13 Extend $\frac{1}{2}$ and $I \rightarrow$ to an endofunctor on *CMS*. $\lrcorner$

The functors $\mathcal{FB}$ and $\mathcal{IF}$ are composed of the above introduced functors. By $\mathcal{P}_{nf}(A)$ and $\mathcal{P}_{n}(A)$ we denote the set of nonempty and finite sets of actions and the set of nonempty sets of actions, respectively.

THEOREM 14 *The endofunctors*

$$\mathcal{FB} = 1 \amalg 2 \cdot (\coprod_{I \in \mathcal{P}_{nf}(A)} (I \rightarrow \tfrac{1}{2}-)) \tag{1}$$

and

$$\mathcal{IF} = 1 \amalg 2 \cdot (\coprod_{I \in \mathcal{P}_{n}(A)} (I \rightarrow \tfrac{1}{2}-)) \tag{2}$$

on CMS have a unique (up to isomorphism) fixed point which is a terminal coalgebra.

PROOF From [AR89, Theorem 5.4] we can derive that the functors—our functor $\coprod$ being the obvious generalization of their $+$ —are locally contractive (see [RT92, Definition 4.2]). Hence, we can conclude from [RT92, Corollary 4.9] that the functors have a unique (up to isomorphism) fixed point which is a terminal coalgebra. $\square$

From the results of [Bar93] we can deduce that the corresponding endofunctors on *Set*—these are obtained by simply forgetting about the metric—also have a terminal coalgebra. We conjecture that similar results can also be obtained in the order-theoretic setting.

Let $\langle X, f\rangle$ be an $\mathcal{FB}$-coalgebra, i.e. X is a complete metric space and $f : X \rightarrow \mathcal{FB}(X)$ is a nonexpansive function. We can view X as a state space. From f we can derive a transition relation and a termination predicate as follows. Consider a state $x \in X$. We distinguish three cases.

* Let $f(x) = \langle 0, 0\rangle$. Then we cannot make a transition from the state x, but we may terminate in x.

* Let $f(x) = \langle 1, I, c\rangle$. The set I consists of the (initial) actions the outgoing transitions of the state x are indexed by. The function $c : I \rightarrow \frac{1}{2}X$ gives us for each initial action a its continuation $c(a)$: the state reached from x by the transition labelled by a. Furthermore, the state x is not a terminating one.

* Let $f(x) = \langle 2, I, c\rangle$. The only difference with the previous case is that we may terminate in the state x.

Note that the obtained system is finitely branching. Furthermore, the system is nondeterministic, i.e. no state has multiple outgoing transitions with the same label. Like in automata theory, one can easily construct for a nondeterministic system a corresponding deterministic one. Similarly, $\mathcal{IF}$-coalgebras can be viewed as image finite systems. The way these systems are described is reminiscent to the interpretation of state machines in [Han97].

THEOREM 15 *$\mathcal{P}_{nk}(A^\infty)$ and $\mathcal{P}_{nc}(A^\infty)$ are a fixed point of (1) and (2), respectively.*

In the rest of this section we prove that $\mathcal{P}_{nk}(A^\infty)$ is a fixed point of (1). The fact that $\mathcal{P}_{nc}(A^\infty)$ is a fixed point of (2) can be shown similarly. Combining Theorem 14 and 15, we can conclude that $\mathcal{P}_{nk}(A^\infty)$ and $\mathcal{P}_{nc}(A^\infty)$ are terminal coalgebras—the result announced in the abstract.

To conclude that $\mathcal{P}_{nk}(A^\infty)$ is a fixed point of (1) we have to show that $\mathcal{P}_{nk}(A^\infty)$ is isomorphic to $\mathcal{FB}(\mathcal{P}_{nk}(A^\infty))$ in *CMS*. For that purpose we introduce the functions e and p,

$$\mathcal{P}_{nk}(A^\infty) \underset{p}{\overset{e}{\rightleftarrows}} \mathcal{FB}(\mathcal{P}_{nk}(A^\infty))$$

show that these functions are arrows of *CMS*, and prove that they form an isomorphism in the category.

DEFINITION 16 The function $e : \mathcal{P}_{nk}(A^\infty) \to \mathcal{FB}(\mathcal{P}_{nk}(A^\infty))$ is defined by

$$e(W) = \begin{cases} \langle 0, 0\rangle & \text{if } W = \{\epsilon\} \\ \langle 1, I, c\rangle & \text{if } \epsilon \notin W \\ \langle 2, I, c\rangle & \text{otherwise,} \end{cases}$$

where

$$I = \{\, a \in A \mid aw \in W \text{ for some } w \in A^\infty \,\}$$

and the function $c : I \to \frac{1}{2}\mathcal{P}_{nk}(A^\infty)$ is given by

$$c(a) = \{w \in A^\infty \mid aw \in W\,\}.$$

The function $p : \mathcal{FB}(\mathcal{P}_{nk}(A^\infty)) \to \mathcal{P}_{nk}(A^\infty)$ is defined by

$$\begin{aligned} p\langle 0, 0\rangle &= \{\epsilon\} \\ p\langle 1, I, c\rangle &= \{\, aw \in A^\infty \mid a \in I \text{ and } w \in c(a)\,\} \\ p\langle 2, I, c\rangle &= \{\, aw \in A^\infty \mid a \in I \text{ and } w \in c(a)\,\} \cup \{\epsilon\}. \end{aligned}$$

$\lrcorner$

EXERCISE 17 Check that the functions e and p are well-defined. $\lrcorner$

Next, we verify that the functions e and p are nonexpansive.

PROPOSITION 18 *The functions e and p are arrows of CMS.*

PROOF We only show that p is nonexpansive. The nonexpansiveness of e can be proved similarly. We only consider the following case. The other cases can be dealt with similarly or are trivial.

$$\begin{aligned} & d_{\mathcal{P}_{nk}(A^\infty)}(p\langle 1, I, c_1\rangle, p\langle 1, I, c_2\rangle) \\ = \;& d_{\mathcal{P}_{nk}(A^\infty)}(\{\, aw \in A^\infty \mid a \in I \text{ and } w \in c_1(a)\,\}, \{\, aw \in A^\infty \mid a \in I \text{ and } w \in c_2(a)\,\}) \\ = \;& \sup_{a \in I} d_{\mathcal{P}_{nk}(A^\infty)}(\{\, aw \in A^\infty \mid w \in c_1(a)\,\}, \{\, aw \in A^\infty \mid w \in c_2(a)\,\}) \\ = \;& \sup_{a \in I} \tfrac{1}{2} \cdot d_{\mathcal{P}_{nk}(A^\infty)}(c_1(a), c_2(a)) \\ = \;& d(\langle 1, I, c_1\rangle, \langle 1, I, c_2\rangle). \end{aligned}$$

$\square$

Showing that $ep = 1_{\mathcal{P}_{nk}(A^\infty)}$ and $pe = 1_{\mathcal{FB}(\mathcal{P}_{nk}(A^\infty))}$ is left as

EXERCISE 19 Verify that e and p form an isomorphism in CMS. $\lrcorner$

References

[Acz88] P. Aczel. *Non-Well-Founded Sets.* Number 14 in CSLI Lecture Notes. Centre for the Study of Languages and Information, Stanford, 1988.

[AR89] P. America and J.J.M.M. Rutten. Solving Reflexive Domain Equations in a Category of Complete Metric Spaces. *Journal of Computer and System Sciences*, 39(3):343–375, December 1989.

[Bai09] R. Baire. Sur la Représentation des Fonctions Discontinues. *Acta Mathematica*, 32(1):97–176, 1909.

[Bar93] M. Barr. Terminal Coalgebras in Well-Founded Set Theory. *Theoretical Computer Science*, 114(2):299–315, June 1993.

[BV96] J.W. de Bakker and E.P. de Vink. *Control Flow Semantics.* Foundations of Computing Series. The MIT Press, Cambridge, 1996.

[BW90] J.W. de Bakker and J.H.A. Warmerdam. Metric Pomset Semantics for a Concurrent Language with Recursion. In I. Guessarian, editor, *Proceedings of the LITP Spring School on Theoretical Computer Science*, volume 469 of *Lecture Notes in Computer Science*, pages 21–49, La Roche Posay, April 1990. Springer-Verlag.

[BW91] J.W. de Bakker and J.H.A. Warmerdam. Four Domains for Concurrency. *Theoretical Computer Science*, 90(1):127–149, November 1991.

[BZ82] J.W. de Bakker and J.I. Zucker. Processes and the Denotational Semantics of Concurrency. *Information and Control*, 54(1/2):70–120, July/August 1982.

[BZ83] J.W. de Bakker and J.I. Zucker. Compactness in Semantics for Merge and Fair Merge. In E. Clarke and D. Kozen, editors, *Proceedings of 4th Workshop on Logics of Programs*, volume 164 of *Lecture Notes in Computer Science*, pages 18–33, Pittsburgh, June 1983. Springer-Verlag.

[Gla90] R.J. van Glabbeek. The Linear Time – Branching Time Spectrum. In J.C.M. Baeten and J.W. Klop, editors, *Proceedings of CONCUR'90*, volume 458 of *Lecture Notes in Computer Science*, pages 278–297, Amsterdam, August 1990. Springer-Verlag.

[Hah32] H. Hahn. *Reelle Funktionen I.* Akademische Verlagsgesellschaft, Leipzig, 1932.

[Han97] P. Hancock. Partial Constructions. Draft, March 1997.

[Hau14] F. Hausdorff. *Grundzüge der Mengenlehre.* Leipzig, 1914.

[HP79] M. Hennessy and G.D. Plotkin. Full Abstraction for a Simple Parallel Programming Language. In J. Bečvář, editor, *Proceedings of 8th Symposium on Mathematical Foundations of Computer Science*, volume 74 of *Lecture Notes in Computer Science*, pages 108–120, Olomouc, September 1979. Springer-Verlag.

[JR97] B. Jacobs and J. Rutten. A Tutorial on (Co)Algebras and (Co)Induction. *Bulletin of the EATCS*, 62, June 1997.

[Kel76] R.M. Keller. Formal Verification of Parallel Programs. *Communications of the ACM*, 19(7):371–384, July 1976.

[Kle56] S.C. Kleene. Representation of Events in Nerve Nets and Finite Automata. In C.E. Shannon and J. McCarthy, editors, *Automata Studies*, volume 34 of *Annals of Mathematics Studies*, pages 3–41. Princeton University Press, Princeton, 1956.

[Kön26] D. König. Sur les Correspondances Multivoques des Ensembles. *Fundamenta Mathematicae*, 8:114–134, 1926.

[Kur56] K. Kuratowski. Sur une Méthode de Métrisation Complète des Certains Espaces d'Ensembles Compacts. *Fundamenta Mathematicae*, 43(1):114–138, 1956.

[Lan69] L.H. Landweber. Decision Problems for ω-Automata. *Mathematical Systems Theory*, 3(4):376–384, 1969.

[Niv79] M. Nivat. Infinite Words, Infinite Trees, Infinite Computations. In J.W. de Bakker and J. van Leeuwen, editors, *Foundations of Computer Science III,* part 2: Languages, Logic, Semantics, volume 109 of *Mathematical Centre Tracts*, pages 3–52. Mathematical Centre, Amsterdam, 1979.

[Plo81] G.D. Plotkin. A Structural Approach to Operational Semantics. Report DAIMI FN-19, Aarhus University, Aarhus, September 1981.

[Pnu85] A. Pnueli. Linear and Branching Structures in the Semantics and Logics of Reactive Systems. In W. Brauer, editor, *Proceedings of the 12th International Colloquium on Automata, Languages and Programming*, volume 194 of *Lecture Notes in Computer Science*, pages 15–32, Nafplion, July 1985. Springer-Verlag.

[RT92] J.J.M.M. Rutten and D. Turi. On the Foundations of Final Semantics: non-standard sets, metric spaces, partial orders. In J.W. de Bakker, W.-P. de Roever, and G. Rozenberg, editors, *Proceedings of the REX Workshop on Semantics: Foundations and Applications*, volume 666 of *Lecture Notes in Computer Science*, pages 477–530, Beekbergen, June 1992. Springer-Verlag.

[RT93] J.J.M.M. Rutten and D. Turi. Initial Algebra and Final Coalgebra Semantics for Concurrency. In J.W. de Bakker, W.-P. de Roever, and G. Rozenberg, editors, *Proceedings of the REX School/Symposium A Decade of Concurrency: reflections and perspectives*, volume 803 of *Lecture Notes in Computer Science*, pages 530–582. Noordwijkerhout, Springer-Verlag, June 1993.

[TJ93] D. Turi and B. Jacobs. On Final Semantics for Applicative and Non-Deterministic Languages. Presented at the 5th Conference on Category Theory and Computer Science, Amsterdam, September 1993.