Università di Pisa

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Technical Report : TR-99-11

A Survey on Obnoxious Facility Location Problems

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April 19, 1999

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Abstract

Obnoxious location models are models in which customers no longer consider the facility desirable and try to have it as close as possible to their own location, but instead avoid the facility and stay away from it. Typical applications are optimal locations of nuclear reactors, garbage dumps, or water purification plants.
This work presents a survey of mathematical models for undesirable location problems in the plane and particularly on networks; solution procedures are briefly described.
A brief review of extensive (obnoxious) facility location problems in networks is also given. Finally critical aspects of existing models are identified and some directions for future search are suggested.

Keywords: survey, obnoxious, undesirable, location.

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1 Introduction

One of the main objectives we attempt to achieve with this survey on Undesirable Facilities Location is to facilitate communication and try to unify notation in this field; so let us first review the main features of Location Analysis.

Location models have been around for a long time. Modern location theory is usually said to have begun with Weber's treatise [68] on the location of industries. For many years hence, his treatment of location models in general and objective functions in particular has dominated the literature. It was Hakimi who first distinguished two kinds of minimizing objectives: *minisum*, the standard efficiency measure and *minimax*, the standard equity measure. Throughout the 1960s and the early to mid 1970s, minisum objectives were applied to private sectors applications, whereas minimax objectives were suggested for public sector applications.

This changed dramatically in the mid 1970s. Goldman and Dearing [26] and Church and Garfinkel [12] seem to have been the firsts to suggest what later became known as *obnoxious location models* (from latin roots "nocere" which means to hurt, and "noxa" which stands for injury), i.e., models in which customers no longer consider the facility desirable and try to have it as close as possible to their own location, but instead avoid the facility and stay away from it. Typical applications are optimal locations of nuclear reactors, garbage dumps, or water purification plants.

Another line of research deals with *fairness* or *equity*: the philosophical background was provided by Rawls [56] who claimed that "there is an analogy between the two principles of justice and the maximin rule for choice under uncertainty".

Often in the literature it is possible to find framework of attracting and repelling forces, similar to those of a magnet: customers or users may either try to attract (or pull) desirable facilities closer to them, or repel (or push) undesirable facilities from them.

2 Elements of location models

A number of attempts has been made to summarize the major components of location models in taxonomies; an excellent survey is the one made by Erkut and Neuman [21] who suggested a very elaborate scheme.

The most important components of location models are (Eiselt and Laporte [18]):

Space. It is customary to distinguish between:

- *continuous* location models, in which the location of facilities in some d-dimensional space $\mathbb{R}^d$ is to be planned
- *discrete* location models, in which facilities may only be located at some prespecified points
- *network* location models. In this case, it may be useful to further distinguish between models in general networks as opposed to specialized networks, like tree and k-tree, as a good number of location problems are $\mathcal{NP}$-hard in general, but solvable in polynomial time on particular graphs.

Furthermore, the space available to facilities to locate may be restricted by the introduction of *forbidden zones*, i.e., areas in which facilities may not be located. Such areas are indeed meaningful as they can model zoning law or other restrictions governing the problem.

The number p of new facilites to be located by the decision maker. Two cases have to be distinguished:

- $p = 1$: in this simplest case only one facility is to be located
- $p > 1$: in this case, it is no longer sufficient to consider only the location aspect of the problem; now it is also necessary decide how to allocate customers to facilities.

When locating multiple new facilities, interactions between them may have to be considered: some models allow interactions between facilities, i.e., terms in the objective include (weighted) distances between new facilities. Examples of reasons for doing so include military installations that are spread out so as to limit the damage by a direct hit, or franchises of gas stations or fast food restaurants that the decision maker tries to disperse so as to avoid direct competition with each other.

The number of existing facilities. Two different types of models exist:

- the number of facilities to be located by the decision maker is fixed

- the number of facilities to be located is variable.

The last type of models is more complicated than the first one because it is necessary to add another variable to the problem, viz. the number of facilities.
Another aspect is that in many location models it is assumed that the decision maker who plans the location of his facilities faces an empty space without any similar or competing facilities, but in general similar facilities already exist and the task is to add new ones in optimal fashion (this problem is known as *conditional location problem*).

The decision maker's objective. A model can be single objective or multiobjective. Among single objective there are at least three different types of objectives:

- minimization of a function of distance: *minimax* or *minisum*
- maximization of a function of distance: *maximin* or *maxisum*
- balancing of functions of distances.

Distance measure. In order to measure distances between customers and facilities or between facilities one of the following metric can be used:

- Minkowski's distances (or l_p distances) which are defined in the plane by

  $$l_p(x, y) = ((x_1 - y_1)^p + (x_2 - y_2)^p)^{1/p} \quad 1 \leq p \leq \infty.$$

  Among the l_p family the most referred distance measures are the Euclidean distance for $p = 2$ and the rectilinear (or Manhattan) distance for $p = 1$.
- network distance.

Distance constraints. In many models there are distance constraints; there are at least two different types of constraints:

- upper bound on distances: "far enough but within reach"
- lower bound on distances: "as close as possible but not too close".

Weights. In the objective function distances can have different weights or equal weights; in the latter case we say the problem is unweighted.

Interactions considered. There are at least three different cases:

- only distances between customers and facilities are relevant, for example locating a number of dump sites in a country
- only distances between facilities are relevant, for example locating missile silos in an unpopulated region
- both distances of the above cases have to be considered, for example locating a number of nuclear facilities in a populated region.

Customers. There are a number of location models in which customers are present: so, it is necessary to know their distribution, their actual demand, as well as customers behaviour. For instance, it may be assumed that customers are either distributed uniformly over a given set, or that they are located at specific points in space or vertices in a network.
As far as customer behaviour is concerned, customers may be assigned to a facility or may be free to choose, in which case the question is whether they always patronize the closest facility or use another utility function.

Objective function. Let us now derive the main categories of objectives: many such classes have been devised (see, for intance, the classification by Eiselt and Laporte [18]). One is the distinction between private and public facilities. As mentioned before, in the literature, private is often associated with *minisum* and public with *minimax*. Reasons for doing so seem compelling: given linear transport cost functions, the minisum objective will locate a facility so as to minimize the sum of transport costs. On the other hand, when locating a public facility such as a swimming pool or a library, one would like to determine a location that is close to even the most distant potential user.
One possibility to choose an objective is to derive it from a set of axioms or value statements. Holzman [32] for example, specifies three "reasonable" axioms and show that the unique objective function that satisfies all of them is the Euclidean distance.
On the other side, considering any existing or planned facility, a user has essentially four choices:

- find it desirable: in this case he would like to have it as close as possible to his own location; the user would like to exert a certain *pull* on the facility
- find it undesirable: he would like to *push* the facility away (NIMBY principle - Not In My Back Yard)

- find it partly desirable and partly undesirable: he would like to have a facilty "close but not too close". Take a supermarket, for instance: on the one hand, customers may wish to live close to the facility for matters of convenience; on the other hand, traffic and noise associated with the supermarket would make this facility undesirable.
  In such a situation a user has essentially three choices:
  - to exert a pulling force with a constraint on the distance between him and the facility: as close as possible but at least as far away as threshold T
  - to exert a pushing force with a distance constraint: as far as possible but within a certain distance
  - to create a mixed *push-pull* objective, for instance a linear convex combination of the two forces
- be indifferent: users who are indifferent can be left out of the model as they will not have any objections either way.

3 Push objectives

Although location theory can be traced back as far as the 17th century, location problems involving undesirable facilities have only been discussed since the early 1970s, but represent one of the major field of research today. One explanation for this imbalance may be that most undesirable facilities are the byproduct of technology and industrialization. Location of nuclear reactors, power plants, dump sites and mega airports are all problems of the 20th century. On the other hand, there have been desirable facilities, such as police stations, hospitals, schools and warehouses for centuries.
First attempts to formalize these problems were made by Church and Garfinkel [12] where *maxian model* was investigated: this model is equivalent to the median, but with a maxisum objective.
Another technique for properly modelling the location of an undesirable facility consists of adding to a minimization problem, lower bounds on the distance between the customer and the facility. Each customer, or cluster of customers, would impose a forbidden zone on the solution space. These models are however less popular than models with maximization objectives and in fact we can only find few papers that deals with a minimization problem with lower bound: Moon and Chaudhry [53] formulate integer programs for the p-median problem

with lower bound between pairs of facilities; see also Moon and Papayanopoulos [54] and Fonseca and Captivo [24]. This case is often referred to as the *semi-obnoxious* problem.
Models of this type later became known as noxious or obnoxious location models, the difference between the two being restricted to semantics rather than objective. Whereas the former refers to potentially dangerous facilities, e.g., those that may be poisonous in case of a leak, the latter type of facilities represents a mere nuisance. In both cases, however, customers try to stay as far away as possible from them.
To visualize the difference between minisum and maxisum objectives, consider a simple example. Let four equally weighted customers be located at the vertices of a square in $\mathbb{R}^2$ with Euclidean distances. The unique minisum location of a single new facility is at the intersection of the diagonals. The maxisum objective, on the other hand, is optimized by locating the facility as far as possible towards infinity, clearly showing the forces that push the facility. In order to avoid such unbounded optimal solutions, it is necessary to define a set within which the facility can be located, something unnecessary with minisum objectives for which it is always better to locate the facility inside the convex hull spanned by the customers. If we were to restrict in the above example the location of a new facility to the convex hull of customers, the maxisum objective would be optimized by locating at any of the four customers locations.

4 Location of undesirable facilities in the plane

4.1 Case $p = 1$

Locating a facility so as to maximize the distance between the single new facility and any of the customers located at fixed points in the plane is referred to as *1-maximin* problem; 1-maximin is equivalent to the geometrical problem of finding the *largest empty circle*, i.e., the largest circle that does not include any given point. The center of such a circle is the optimal location for such an undesirable facility.
Most of papers dealing with the Euclidean distance measure use the Karush-Kuhn-Tucker (KKT) conditions, and enumerate all local optima to find a global optimum.
On the other hand, the most popular method to solve the rectilinear

1-maximin problem is based on solving a sequence of linear programs (LPs) for segments of the feasible region; see Drezner and Wesolowsky [15], Melachrinoudis [46], Mehrez *et al.* [44].
A good number of papers (Dasarathy and White [13], Drezner and Wesolowsky [14], Melachrinoudis [45]), regardless of the metric, use geometric notions and suggest simple graphical solution procedures involving growing circles (as far as Euclidean distance is concerned) around existing facility points or customers, and selecting the final point not covered by any circle. This graphical method is not limited to Euclidean metric. For example, in the case of rectilinear distances, the procedure grows diamonds instead of circles. All l_p-norms can, in fact, be used with this procedure.
In the field of 1-maximin problems the main results are:

Dasarathy and White [13] solve the unweighted 1-maximin problem in a bounded, convex polyhedron in $\mathbb{R}^d$ with Euclidean distances. The authors show that the objective of the problem is not convex and may have local optima. They also determine a finite set of candidates for the optimal solution using the KKT conditions and propose an algorithm.

Drezner and Wesolowsky [14] deal with the 1-maximin problem with weighted Euclidean distances (used to reflect the relative importance of existing sites) and subject to constraints that guarantee that all facility-customer distances are away, but within reach; the feasible region is then composed of the intersection of circles each centered at a customer point. A numerical bisection search procedure is given.

Melachrinoudis [45] solves the 1-maximin problem with Euclidean distances in the presence of constraints that limit the location to a bounded region.

Melachrinoudis and Cullinane [47] presents an extension of the problem in [45] in which not only does the facility have to be inside the bounded region, but it must also be at a certain minimum distance from the customers, i.e., there are circular forbidden regions around customers.

Karkazis and Karagiorgis [37] extend the formulation given in [47] to a region containing a set of areas defined as either protected or forbidden and not necessarily circular in nature.

Melachrinoudis and Cullinane [49] investigate the problem of locating a single facility in the plane so as to minimize the maximum weighted inverse square distance from the facility to n given points, the customers. The objective is justified as various types of pollution

such as noise and radiation, tend to decrease with the square of the distance between polluting facility and customer.
The region in which the facility is to be located, is a bounded set of points in two-dimensional Euclidean space and contains the n points with which the facility to be located will interact. The authors demonstrate that (i) the local minimun is attained either in the convex hull of these given points, or (ii) on the boundary of the feasible region. For the special case the feasible region is a convex polygon, the authors describe an $O(n^4)$ algorithm. This problem is equivalent to the maximization of the minimum weighted square distance [23].

Melachrinoudis and Cullinane [48] state some properties of the optimal solution of the rectilinear weighted 1-maximin problem on a convex polygon S, which effectively limit the search for optimal solutions to a subset of S. The algorithm is based on a search over all local maxima, similar to the algorithm for the Euclidean distances. [48] also provide a survey of results concerning maximin and maxisum objectives with Euclidean and rectilinear distances.

Karkazis [34] considers a model without customers. He requires again that the single facility must be located in a polygonal region, but protected areas exist in which the facility may not be located. The objective is to maximize the minimum distance between the facility and any point in one of the prespecified protected regions. Again this problem reduces to the geometric problem of finding a circle of maximum radius with its centre in the permissible region, so that the circle does not intersect any of the protected regions such as nature preserves, parks and so on.

On the other hand the problem of locating a facility so as to maximize the sum of distances between a single new facility and all of the customers is referred to as *1-maxisum* problem.
We observe that maximin objective, being concerned with the minimum distance from existing facilities (customers for example), seems to be better suited than the maxisum objective when locating undesirable facilities. The maxisum objective, being concerned with the sum of distances, may result in a solution where the optimal location is in the immediate neighbourhood of an existing facility. Alternatively, the maxisum objective can be useful in solving a multiobjective problem.
As far as 1-maxisum problem is concerned the main results are:

Hansen *et al.* [31] consider the 1-maxisum problem in the plane.

By using a decreasing nonlinear function of distance, however, they actually formulate the problem using a minisum criterion. They give a theorem which limits the search for optimal solutions to the vertices of the convex hull of the feasible region. They also provide a geometrical solution techique, which they term *Big Square - Small Square* (BSSS) method.
The BSSS method is a continuous Branch and Bound (B&B) procedure proposed by Hansen *et al.* [31] and modified by Plastria [55] whose success is due to its simplicity and versatility to solve planar problems: it starts with a region known to contain an optimal solution for the problem to be solved, which is dynamically divided into smaller regions, discarding from further considerations those checked to be infeasible or non promising.

Hansen *et al.* [30] solves the 1-maxisum problem using analytical results that limit the search to a subset of the feasible region: they propose a geometrical B&B algorithm.

Plastria [55] extends the B&B proposed in [30] for general single facility problems, with distances measured by mixed norms and with any continuous function of distances as an objective. The method caters for forbidden regions such as rivers or lakes, and is proposed as a site generation tool.

Kaiser and Morin [33] reduces a number of "reasonable" objectives in the context of undesirable facilities to the maximization of the sum of the logarithms of Euclidean distances.

As a step towards more realistic models Buchanan and Wesolowsky [5] propose a graphical approach for locating an undesirable facility in the plane using general asymmetric distances.

Romero-Morales *et al.* [59] address the problem of locating a semi-obnoxious facility in a planar feasible region given by a finite union of polygonal areas, in which not only transportation costs, but also the environmental impact in population centers is important. In particular the environmental cost is modeled as a nonincreasing convex function of the distances between the facility and the population center, whilst the transportation cost can be better modeled as a Lipschitz-continuous nondecreasing function of distances. The solution method proposed is the well known BSSS, with a new bounding scheme which exploits the structure of the problem based on Lagrangian relaxation techiques.

4.2 Case $p > 1$

In this field the main results are:

Hansen *et al.* [31] define *anti-Weber* and *anti-Rawls* problems as the counterparts of *p-Weber* and *p-Rawls* problems in the plane: the first problem is solved with a B&B method similar to the BSSS method. The anti-Rawls problem is solved with a simple graphical method.

Drezner and Wesolowsky [16] study the problem of locating several obnoxious facilities simultaneously in continuous space. In particular they propose two model formulations: in the first formulation they minimize the maxisum weighted distance in the system subject to constraints which require that the distances between the facilities and fixed points (population centers or other installations) must exceed specified values: we refer to this formulation as the minimax formulation. In the second formulation, the smallest weighted facility-to-fixed-point distance in the system must be maximized, given that every fixed point must be within reach of the closest facility: we refer to this formulation as the maximin formulation.
These two problems are closely linked by a sort of duality: in other words, a certain type of constrained minimax location problem is dual to the maximin location problem. The main duality results are:

- for a fixed value f_0, the set of all feasible potential locations ($S_P(f_0)$) in minimax problem for which the value of objective function is not bigger than f_0, is equal to the set of all feasible potential locations ($S_D(f_0)$) in the maximin formulation for which the value of objective function is not smaller than f_0; i.e., $S_P(f_0) = S_D(f_0)$ where P stands for primal and D stands for dual
- if the minimax and maximin problems are not trivial (i.e., if their optimal solutions are constrained by at least one constraint) the optimal solution for the primal is also optimal for the corresponding dual and viceversa.

So this minimax problem may be useful in itself or may be used as an aid to the solution of the original maximin problem.
In general, the two dimensional location problem is difficult due to the combinatorial problems arising from the need for allocating fixed points to facilities in the model.
However, Drezner and Wesolowsky [16] present an efficient method for solving the problem when facilities and fixed points are restricted

to a line: finding a feasible solution to the one-dimensional problem is, in effect, a set covering problem in that each of the fixed point must be covered by at least one facility and furthermore p facilities must be opened. We observe that the general set covering problem is $\mathcal{NP}$-hard, but this special case is relatively easy due to a particular property of the underlying matrix (the *continuity property*).
The complexity of the proposed procedure is $O(np)$ where n is the number of fixed points, in addition to the initialization step of $O(n^3)$. The principle underlying their method of finding the optimal solution is that of eliminating sites which cannot form part of a solution better than one already obtained. The complexity of the algorithm is bounded by $\min\{O(n^3), O(n^2p)\}$ as the above procedure to find a feasible solution is repeated $O(n)$ times.

Erkut *et al.* [20] show that the p-maxian is $\mathcal{NP}$-hard and device an exact Branch&Bound method as well as a heuristic method that finds optimal solutions most of the time.

5 Location of undesirable facilities on networks

5.1 Case $p = 1$

In this section 1-maximin and 1-maxisum problems will be addressed. Let us first introduce some notations.
Let $G = (V, E)$ be an undirected graph with vertex set $V = \{v_1, \ldots, v_n\}$ (the set of existing facilities or customers) and edge set E. Let $A(G)$ denote the continuum set of points on the edges of G; $A(G)$ is thus the set of candidate points for the location of the undesirable facility and consists of the vertices V and infinite points on the edges.
For $y \in A(G)$ let $d(v_i, y)$ denote the lenght of a shortest path from vertex v_i to y and let $\alpha_i \geq 0$ be a weight attached to vertex v_i.
The 1-maximin problem can thus be formulated as

$$\text{Max } \min_i \alpha_i d(v_i, y).$$

There is almost no literature on 1-maximin problem on a network. This is understandable, since the unweighted problem is trivial on a network. There is always an optimal location, halfway on a longest edge of the network. The weighted problem can also be solved easily by considering all pairs of vertices.

Burkard *et al.* [6] derive algorithms with linear running time for the cases when G is a path or a star, thus improving previous results ($O(n \log n)$) of Tamir [63]. They also treat extended stars, which can be obtained from stars by subdividing edges and introducing additional vertices on them; on a such graph with b branches, 1-maximin problem can be solved in $O(n + b \log n)$ time. For general trees, Burkard *et al.* [6] improve an algorithm of Tamir [63] by a factor of $\log n$, obtaining an algorithm of complexity $O(kn \log n)$ where k is the diameter of the tree.
A linear algorithm for the unweighted center problem on an arbitrary tree with neutral and obnoxious vertices is also described.

The problem of locating a facility so as to maximize the weighted sum of distances (determined as the shortest paths) from a facility to all customers located at the vertices of the network is referred to as the *1-maxian* or *antimedian* problem or as the 1-maxisum problem. The 1-maxisum problem can thus be formulated as

$$\text{Max} \sum_{i=1}^{n} \alpha_i d(v_i, y).$$

In this field the main results are:

Church and Garfinkel [12] consider the weighted maxisum problem on a general network and show that the search for the optimal solution can be limited to a finite set of points on the network, which is made up of the *dangling* or *pendant* vertices (i.e., vertices of degree one) plus a finite set of *bottleneck points.* These bottleneck points, associated with the cycles of the network, are points linked to some inhabitant's location by two distinct shortest paths and are the points at which local optima of the distance function occur. Trivially, if the network is a tree, an optimal point is a dangling vertex.
Dealing with this finite set of candidate points leads to a search algorithm that requires at most an $O(n^3)$ effort where n is the number of vertices of the network.

Ting [66] devices a linear time algorithm for 1-maxian on trees.

Minieka [51] investigates anticenter and antimedian problems; an *anticenter* is a point on the network maximizing the farthest distance from the facility to a customer. Minieka in [51] studies the unweighted version of these two problems on directed networks. As far as 1-maxisum is concerned the set of possible sites for the antimedian is composed of vertices and specific points on directed and undirected arcs similar to the bottleneck points used by Church and Garfinkel [12].

Berman *et al.* [3] deal with the minimum covering criterion for obnoxious facility location on a network. The objective of their work is to find a location for a new facility on any part of the network (edge or vertex) such that the number of vertices within a prespecified distance is minimized. In more general cases, there is a different weight for each vertex and the objective is to minimize the total weight within the prespecified distance from the new facility (see also Sung and Joo [62]). The paper includes an analysis of the problem, identification of special cases where the problem is easily solved, an algorithm to solve the problem in general based on identifying optimal segment on each edge, and a sensitivity analysis of the covering distance.
It should be noted that the problem is trivial iff the longest edge in the network is at least twice the covering distance long; in this case the optimal solution consists of locating the new facility at the middle point of that edge and has null cost.

Carrizosa and Plastria [8] address the problem of locating an undesirable facility within a compact set by minimizing a strictly decreasing bounded below function of the squared Euclidean distances to a set of fixed points (the existing facilities or cities). Their approach consists of solving a sequence of maxmin problems by using generalized cutting planes.

Another way to tackle an obnoxious facility location problem on a network is to consider a procedure in which inhabitants, located on vertices of the network, vote on the choice of location; such a referendum procedure is frequently used in Switzerland to decide on location of public service.
The voting solution is then called the *anti-Condorcet point* and is such that no other point is farther from a strict majority of inhabitants. Labbé [42] shows that on general network with an odd number of inhabitants (generally different from the number of vertices n), there exists a finite set of points, made by bottleneck points and pendant vertices, containing all such solutions.
Labbé [42] also shows that this result does not directly extend to an even number of inhabitants on a general network.
In the special case of a tree network, one of the extreme vertices of a diameter (path of maximum lenght) is an anti-Condorcet point and a

linear algorithm for finding it is presented.
Finally [42] makes a comparison between the anti-Condorcet point and the antimedian and provides a best possible bound on the maximum decrease of the total distance to the inhabitants when a voting solution is preferred to a planning one.

Stowers and Palekar [61] propose a model that simultaneously considers the risk posed by location and transportation risks while searching for an optimal location on a network. The location of the new facility is not restricted to some known set of potential sites; it is selected so as to minimize the impact on the area that will be affected by the new site, i.e. the area around the site as well as those areas that will be impacted by the traffic due to the obnoxious facility.
Special cases of the problem that admit combinatorial solutions are identified: if only travel exposure is considered, an optimal solution can be found by considering only the set of vertices. For the case of population concentrations at vertices only, a discrete set that includes at least one optimal solution has been identified: such set is called a *finite dominating set* (FDS).
Finally, for uniformly distributed populations along edges and population concentrations at vertices, Stowers and Palekar [61] show that it is possible partition the edges of the network and search for the global optimum solution.

5.2 Case $p > 1$

Some definitions for the multifacility case follow.
Let $G = (V, E)$ be a graph as defined in previous section. All problems of this section, unless otherwise specified, consist of finding a set of p points $\{y_1, \dots, y_p\}$ in $A(G)$, such that a certain objective function is maximized: this means that the p new facilities or suppliers, can be located at the vertices or in the interiors of edges as well.
The objective function of these location problems involves distances between pairs of facilities: each distance value is a weighted distance between some new facility and existing facility, e.g., $\alpha_{ij} d(v_i, y_j)$, or between a pair of new facilities, e.g., $\beta_{jk} d(y_j, y_k)$: α_{ij} with $i \in \{1, \dots, n\}$ and $j \in \{1, \dots, p\}$ and β_{jk} with $j, k \in \{1, \dots, p\}$ are thus nonnegative weights standing for the coefficients of interaction between facilities.
The main results for the multifacility case follow.

Moon and Chaudhry [53] discuss and classify models with push objectives. Two main categories are outlined: one in which the distances between new facilities are constrained (relevant when a decision maker locates multiple facilities or when some facilities already exist), and one which constrains distances between facilities and customers. Some complexity results are surveyed.

The problem of locating p new facilities on a network without customers so as to maximize the minimum interfacility distance (measured by shortest paths) is referred to as *p-dispersion-min* problem and belongs to the class of p-maximinmin problems. Military and civil applications exist for this model.
Using the notation introduced above, the p-dispersion-min problem can be formulated as

$$\text{Max}\left\{\min_{\substack{jk\\ j\neq k}} \beta_{jk} d(y_j, y_k)\right\}.$$

As far as this problem is concerned the main results are:

Shier [60] proves that on tree networks the following relation holds:

$$\frac{1}{2} z((p+1)\text{-dispersion-min}) = z(p\text{-center})$$

where $z(\cdot)$ denotes the objective value of problem $(\cdot)$.
This is an interesting duality result between problems belonging to different classes, one being a pull and the other a push objective.

Kuby [41] show that in general networks the following relation holds:

$$\frac{1}{2} z((p+1)\text{-dispersion-min}) \leq z(p\text{-center}).$$

By this duality result Chandrasekaran and Daughety [9] develop two very similar polynomial ($O(n^2 \log p \log n)$) algorithms for the p-dispersion-min and $(p-1)$-center problems on trees. For the center problem they solve a series of *anticover* problems: the anticover problem locates the maximum number of facilities with the restriction that no two are closer than a specified distance from each other.

Chaudhry *et al.* [10] discuss anticovering problem on general network. The problem, which is equivalent to finding the maximum independent set, is then solved by a heuristic method.

Erkut [19] shows that the p-dispersion-min problem is $\mathcal{NP}$-hard (via transformation from CLIQUE) and presents an exact B&B method and a heuristic algorithm.

Kincaid [39] describes an implementation of Simulated Annealing and Tabu Search for the unweighted discrete version of the p-dispersion-min problem where each variable y_j can take a value from a finite set of candidate locations. Computational results show that Tabu Search performs better than Simulated Annealing or the greedy heuristic by Erkut and Neumann [22].

Erkut [19] also addresses the *p-anticenter dispersion* problem, which belongs to the class of *p-maximinmin* problems. The objective is to locate the new facilities far from each other and also far from customers. Although a generalization of the p-dispersion problem, this problem can be solved more efficiently, using a minor modification of the B&B method developed for the p-dispersion problem.
The p-anticenter dispersion problem consists of

$$\text{Max } \min\left\{ \min_{ij} \alpha_{ij} d(v_i, y_j);\ \min_{\substack{jk \\ j\neq k}} \beta_{jk} d(y_j, y_k) \right\}. \tag{1}$$

We should observe that this problem subsumes, as particular case in which $\alpha_{ij} = 0$ for all i and j, the p-dispersion-min problem above mentioned.

Tamir [64] considers obnoxious facilities location on graphs using the p-maximin and p-maxisum criteria.
As far as the p-maximinmin (or p-anticenter dispersion) problem is concerned, the formulation used by Tamir is the one given by equation (1).
On the other hand the p-maxisummin criterion is defined by the following objective function

$$\text{Max } \left\{ \sum_{i=1}^{n} \sum_{j=1}^{p} \alpha_{ij} d(v_i, y_j) + \sum_{j=1}^{p} \sum_{k=1}^{p} \beta_{jk} d(y_j, y_k) \right\}. \tag{2}$$

By the homogeneous versions of the above models we refer to the case where the new facilities $\{y_1, \dots, y_p\}$ provide and receive identical services (formally $\alpha_{ij} = \alpha_i \ \forall i, j$ and $\beta_{j,k} = 1 \ \forall j, k$).
For the general (nonhomogeneous) case Tamir [64] shows that both models are strongly $\mathcal{NP}$-hard even when the underlying graph consists of a single edge.
As far as p-maximinmin problem is concerned other main results [64] are the following:

- the p-maximinmin problem is $\mathcal{NP}$-hard when defined on a general graph even when only distances between new facilities are considered, i.e., when restricted to the p-dispersion problem, as shown in [19].
  Note that if each facility is further restricted to a vertex, then the resulting discrete p-maximinmin model generalizes the independent set problem [25]. Therefore, the discrete version is also $\mathcal{NP}$-hard
- unless $\mathcal{P}= \mathcal{NP}$ there is no polynomial ε-approximation algorithm for the homogeneous p-maximinmin problem on a general graph with $\varepsilon > 2/3$. A 1/2-approximation for the same model is given by the following polynomial greedy heuristic: construct a sequence of p points such that each point is as far apart as possible from the set of points selected before. This heuristic is motivated by the 2-approximation procedure for the p-minimaxmin model given by Dyer and Frieze in [17]
- turning to tree graphs Tamir [64] presents a linear time algorithm for the (homogeneous) *r-anticover* problem and applies it to get polynomial time algorithms for the homogeneous p-maximinmin problem. For example, it is possible to solve the single facility case in $O(n \log^2 n)$ time, improving upon previous results. The r-anticover problem consists of finding the maximum number of facilities such that the distance between any two of them is greater or equal than r.

As far as p-maxisummin problem is concerned the main results [64] are:

- even the homogeneous case is strongly $\mathcal{NP}$-hard on general graphs
- as far as the homogeneous case on tree graphs is concerned, Tamir [64] presents an $O(np)$ dynamic programming scheme for its solution. Furthermore by using concavity properties the model can be reformulated as a maximum concave separable quadratic flow problem. This formulation leads to polynomial and strongly polynomial algorithms for the homogeneous p-maxisummin problem.

Another two interesting problems are the *p-defense-min* problem where the objective is to maximize the sum of distances between each facility and its nearest neighbor (also this problem belongs to the class of the *p-maxisummin* problems), and the *p-maximinsum* problem in which the decision maker maximizes the smallest sum of distances between any of the facilities and their customers. Customers are first allocated to facilities and the sum of shortest facility-customer distances is determined for each facility. The smallest such sum is to be made as large as possible.

The p-defense-min problem has been introduced and motivated by Moon and Chaudhry [53]; consists of finding points $\{y_1, \dots, y_p\}$ in $A(G)$ and has the following objective:

$$\text{Max} \left\{ \sum_{j=1}^{p} \min_{\substack{k \\ k \neq j}} \{ d(y_j, y_k) \} \right\}.$$

Given a finite set of points $Y = \{y_1, \dots, y_p\}$ in $A(G)$ and taking into account also facility-customer interaction, the discrete *p-defense-min* problem has also been formulated by Erkut and Neuman [22] as

$$\text{Max} \sum_{j=1}^{p} \min \left\{ \min_{i} \{ \alpha_{ij} d(v_i, y_j) \}; \min_{\substack{k \\ k \neq j}} \beta_{jk} d(y_j, y_k) \} \right\}. \tag{3}$$

We should observe that this problem subsumes, as particular case in which $\alpha_{ij} = 0$ for all i and j, and $\beta_{jk} = 1$ for all j and k, the p-defense-min problem above mentioned.

Both models stated above (p-maxisummin and p-maximinsum) are $\mathcal{NP}$-hard when defined on general graphs, and the same result holds even when the discrete versions are considered, in which the points $\{y_1, \dots, y_p\}$ are confined to the vertex set of the underlying graph.
Turning to tree graphs, the recursive solution approach of Tamir [64] seems to be applicable for above problems. However, this approach is discrete in nature, and therefore it requires that optimal points belong to some prespecified discrete set of points of polynomial cardinality. This latter request is necessary in order to have a solution procedure of polynomial complexity. If such a set is identified it is sufficient to reduce the model to its discrete version by augmenting the points in this set to the vertex set of the tree. Indeed Tamir [64] constructs polynomial algorithms of complexity $O(p^2n^3)$ and $O(p^2n^2)$ for the discrete versions of the two above problems, respectively. Therefore, to obtain polynomial procedures for solving the continuous version it is necessary to identify a finite dominating set.

A related problem, the unweighted discrete *p-defense-sum* problem is studied in [39] by Kincaid. It can be formulated as

$$\text{Max}\ \left\{\sum_{j=1}^{p}\sum_{k=1}^{p} d(y_j, y_k)\right\}$$

where each variable y_j can take a value from a finite set of candidate locations.
This problem is shown to be strongly $\mathcal{NP}$-hard by Hansen [29] but it can be solved in polynomial time when restricted to trees (see Chandrasekaran and Daughety [9] and Hansen [29]).
Kincaid describes an implementation of Tabu Search and Simulated Annealing as for the p-dispersion-min problem and shows that the first heuristic performs better than the latter. Computational results show that the p-defense-sum problem is easier to solve than the p-dispersion-min problem.

Another p-facilities problem has been suggested by Ting [67] and consists of finding points $\{y_1, \ldots, y_p\}$ in A(G) with the following objective:

$$\text{Max}\ \left\{\sum_{i=1}^{n} \alpha_i \min_j \{d(v_i, y_j)\} + \sum_{j=1}^{p}\sum_{k=1}^{p} d(y_j, y_k)\right\}.$$

Following the rapid technological advancements in hardware and software, research efforts have shifted towards more realistic formulations over the last 10 years. Models where distances are measured in the Euclidean or rectilinear metric are not adequate for most real life problems. The spread of pollution, for instance, is usually not symmetric: in the case of prevalent winds, pollution may spread further in one direction than in others and that can vary from time to time. An algorithmic scheme for locating facilities causing airborne pollution, given a detailed description of the meteorological conditions, is developed by Karkazis and Boffey [35] and by Karkazis *et al.* [36] who recommend using an interpolation scheme to estimate the data between the discrete wind directions. Karkazis and Papadimitriou [38] on the other hand, consider the same problem in the plane. The objective proposed is the minimisation of the sum of weighted risk factors for each vertex summed over all possible wind directions.

Klein and Kincaid [40] deal with the discrete *p-anticenter* problem, that consists of finding p facility locations which will maximize the minimum distance between demand points (or existing facilities or cities) and their respective nearest facilities.
They state the discrete p-dispersion problem as follows.
Let $G = (V, E)$ be a finite connected undirected graph and let (R, N) be a partition of V where R denotes the set of existing facilities or cities (with $|R| = m$) and N denotes the set of potential obnoxious facility location sites (with $|N| = n$). The assumption that R and N are disjoint implies that a city would never be considered as a potential location for an obnoxious facility: if a vertex was in R and N, it would be an unattractive option since the value of objective function would equal zero, if this facility site was choosen.
The discrete p-anticenter problem can thus be formulated as

$$\text{Max}\left\{\min_{i\in R}\min_{j\in N}\alpha_i d_{ij} y_j \;\; \text{s.t.} \;\; \sum_{j\in N} y_j = p,\; y \in \{0,1\}^{|N|}\right\}$$

where d_{ij} is the shortest path distance between the city i and the facility located in site j, α_i is a nonnegative weight used to express the relative disutility between the city i and a possible new facility and y_j is the decision variable relative to facility j (equal to 1 if a facility is located in site j and 0 otherwise).
Given the matrix D of the weighted shortest path distances (D can be constructed in $O(n(n+m)^2)$ time or simply $O(nm^2)$ time if we assume $n \ll m$) the algorithm given in [40] consists of determine the smallest entry in each of n columns, i.e., the closest city to each potential facility site ($O(mn)$ time) and sort these values ($O(n \log n)$): an optimal solution is obtained opening the p facilities corresponding to the last p values in the ordering. The overall complexity of the algorithm is thus dominated by the calculation of shortest path distances and is $O(nm^2)$. It should be observed that to date this is the only known discrete location problem on a general network solvable in polynomial time even though the discrete p-center problem is known to be $\mathcal{NP}$-complete.

Labbé *et al.* [43] deal with the discrete *repulsive p-median* problem which consists of finding p facility locations which will maximize the sum of distances between each existing facility or city and the nearest obnoxious facility.
Let $G = (V, E)$ be a complete and euclidean graph and let (R, N) be

a partition of V where R denotes the set of cities and N denotes the set of potential obnoxious facility location sites; the problem can thus be formulated as

$$\text{Max}\left\{\sum_{i\in R}\min_{j\in N} d_{ij}y_j \;\text{ s.t. }\; \sum_{j\in N} y_j = p,\; y\in\{0,1\}^{|N|}\right\}$$

where d_{ij} is the euclidean distance between the city i and the facility located in site j, and y_j is the decision variable relative to facility j. Labbé, Ndiaye and Maffioli ?? show that the repulsive p-median problem is $\mathcal{NP}$-complete.
They propose several families of cutting planes and local search based heuristics to solve the problem. A branch and cut approach is under development.

Chhajed and Lowe [11] consider a generic discrete multifacility location problem (GMLP) in which p new facilities are to be located. The locations of these new facilities are represented by decision variables $Y = \{y_1, \ldots, y_p\}$. Each y_j can take a value from the set $Z = \{z_1, \ldots, z_w\}$. Demands for services from the new facilities are located at points (called existing facilities) denoted by $v_1, \ldots, v_n$.
Although the examples of generic problem cited in [11] refer to networks, this need not be the case.
Several $\mathcal{NP}$-hard location problems are instances of GMLP stated by [11]; among the maximization ones there are:

- p-maxisummin problem (Erkut *et al.* [20], Tamir [64]), given by equation (2).
  This problem with $\alpha_{ij} = 0$ for all i and j is the p-defense-sum problem
- p-anticenter dispersion problem, (Erkut [19], Tamir [64]), given by equation (1) which, as before observed, has the p-dispersion-min problem as a particular case
- *p-dispersion-sum* problem (Erkut and Neuman [22]), that is

$$\text{Max}\;\min_j\left\{\sum_i \alpha_{ij} d(v_i, y_j) + \sum_k \beta_{jk} d(y_j, y_k)\right\}$$

- p-defense-min problem (Erkut and Neuman [22]), given by equation (3).

Chhajed and Lowe [11] present a unified algorithm which solves all location problems that fit in GMLP formulation when the *dependency graph*, defined by interactions between pairs of facilities, is a *k-tree* for a fixed value of k.
Let us now give the definition of k-tree.
A k-clique is a complete graph on k vertices. A k-tree is recursively defined as follows: a k-clique is a k-tree. Given a k-tree and a subgraph of the k-tree which is a k-clique, the graph obtained by introducing a new vertex and connecting it to every vertex of the k-clique is again a k-tree.
k-trees form a rich class of graphs and contain several well known graph families.
When no particular relations between the facilities exist, GMLP is a non-serial dynamic programming problem and can be solved efficiently by successive variable elimination. The complexity of the proposed algorithm, when the dependency graph is a k-tree is $O(w^{k+1}(kp+|B|))$ where w is the number of candidate locations and $|B|$ is the number of functions involved in the GMLP formulation.
Although efficient algorithms exist for several problems when restricted to k-trees, recognizing whether a graph is a k-tree remains a difficult problem; an efficient characterization for 2-tree is known, whilst, for a fixed value of k, recognizing k-trees can be done in $O(n^{k+2})$ where n is the number of vertices.

Welch and Salhi [69] deal with the obnoxious p facilities network location problem involving weighted Euclidean distances and propose three heuristics to solve the maximin formulation considering a pollution dispersion equation and facility interaction.
A heuristic procedure is first introduced for solving the single facility network location problem which consists of building up polygons that model pollution spread about the vertices of the network. In particular for any given weighted distance, a dispersion map P_i can be formed around the vertex i of the network: a series of dispersion maps is thus obtained repeating this step for all existing vertices. The points of equal or higher pollution are defined by the convex hull of P_i and the arc lenghts containing those arc points that do not lie within any convex hull, correspond to the feasible solution intervals for the weighted distance choosen. An iterative procedure is then evoked. This procedure serves as a basis for the development of the heuristics which locate p facilities.

As far as p facilities problem is concerned the following three heuristics are proposed:

- a heuristic procedure based on that given previously, taking into account the facility interaction. A step-by-step placement of the facilities is applied: each facility will be sited in turn and the infeasible parts of the so far feasible solution intervals removed. Then at each iteration the procedure employes a reduction of the continuous feasible location set to that of a discrete set of points. This will continue until all facilities have been sited or has been proven that for a given value of weighted distance there exists no feasible siting solution
- a heuristic which combines both p-maximin and p-maxisum objectives in a lexicographic manner. This idea comes from the observation that often in Facility Location a number of facilities can be relocated, without violating the feasibility constraint or reducing the weighted distance. This implies that certain facilities may be resited to improve the solution of a p-maxisum formulation, whilst maintaining the bound set by the p-maximin problem
- a heuristic based on Simulated Annealing: in the two heuristics above mentioned for the p facilities problem, each feasible interval can be approximated by a finite set of points, but using a Simulated Annealing technique it is possible to retain the entire lenght of each interval.

The proposed heuristics are evaluated for up to six facilities on a set of randomly generated networks having 20 to 40 vertices and low or medium arc density.

Finally, it should be observed that aside from dispersion and defense problems, there is a severe lack of research on multifacility location problems with a maximization objective. This is a major deficiency in the location area and it is probably not a result of oversight, but of avoidance: even the simplest of these multifacility problems turns out to be $\mathcal{NP}$-complete.

6 A brief review of extensive facility location in networks

Mesa and Boffey [50] present a comprehensive review of *extensive* facility location in networks; a facility is called extensive if, for purposes of location, it is too large in relation to its environment regarding the activities of interest for it to be considered as a point.
Transportation of hazardous materials is an example of extensive facility location: hazardous material, such as nuclear waste or a toxic chemical, is to be transported from a source to a disposal site. It is required to find a path from source to destination which is a tradeoff between the minimization of transportation cost and the minimization of risk, due to possible accident, to the population living or working nearby. This problem is stochastic with respect to the occurrence of accidents.
The review by Mesa and Boffey [50] considers only problems which involve interaction with the environment in some way. They propose two main categories of papers according to whether or not the facility is "free to move around": the two cases are referred to as *end-point free* and *end-point fixed* respectively. Inside each main group they further distinguish according to the following schema:

- the end-point free case is further subdivided into
  - *unconstrained* problems, i.e. there are no restrictions on the facility
  - *constrained* problem, i.e. the facility is *intrinsically* restricted in some way other than by fixed end point
- the end-point fixed case is further subdivided into
  - *central* problems, i.e. the facility is essentially attractive in nature
  - *anticentral* problems, i.e. the facility is essentially repulsive in nature.

In almost all of the end-point free unconstrained problems, facilities are central in nature, the reason for this being that without some restraints an anticentral facility degenerates to a point or set of points. The only exception is in Tamir and Lowe [65] who search for an optimal p-forest in a tree employing an objective which is a linear combination of cost functions such as lenght and distancesum, and propose a polynomial time algorithm ($O(n^2p^2)$ where n is the number of vertices).

Among the end-point free constrained problems, Minieka [52], Richey [58], Aneja and Nair [1] and Hakimi *et al.* [28] consider anticentral facilities as well as central ones. Much of the work associated with these papers is complexity oriented. However, among papers mentioned above, the most comprehensive one is that of Hakimi *et al.* [28] who considered 64 different problems only a few of which had received attention previously. These problems arise by choosing one possibility from each of the following six pairs:

1. *underlying structure*: tree or general graph
2. *shape of facilities*: path or tree
3. *structural extremities*: vertices or not necessarily vertices
4. *decision criteria*: eccentricity or distancesum (respectively maximin or maxisum criteria in anticentral facilities)
5. *type of facility*: central or anticentral
6. *number p of facilities*: $p = 1$ or $p > 1$.

It was found that there are more polynomial time solvable problems among those using the eccentricity than among those that have distancesum as objective. Generally, problems on arbitrary graphs are $\mathcal{NP}$-hard, but in all cases the computational compexity character ($\mathcal{NP}$ or polynomial) is the same for single and multiple problems, except when p is not fixed.

The end-point fixed problems are more practically oriented and the underlying graph structure is usually taken to be a transportation network: these problems are frequently concerned with flow from fixed origins to fixed destinations and as anticentral problems are concerned, the main application is the transportation of hazardous materials. The basic cause of risk results from the possibility of a vehicle being involved in an accident followed by release of the hazardous material carried. When the material does not spread widely then it is sufficient to choose routes across not very populated areas as shown by ReVelle *et al.* in [57]; it is also possible to incorporate arc weights to take into account different accidents possibilities, Batta and Chiu [2], Gopalan *et al.* [27]. The situation is similar when explosive material is being carried since the effects of an explosion will be confined to at most a range of a few hundred metres. On the other hand, some materials are volatile at ordinary temperatures and so, after release,

could be borne a considerable distance by wind. Modelling this situation requires that account should be taken of the frequencies of wind directions and speed, see Boffey and Karkazis [4].
Another aspect of routing hazardous materials, that is attracting increasing attention, is that of *equity* which is largely discussed in Mesa and Boffey [50] and related works.
It is noteworthy that situations in which the environment impacts on a vehicle traversing a route (such as routing in regions of terroristic activities and the routing of security vans) belong to anticentral end-point fixed class of problems too.

7 Some critical points of view

Many of the analytical models discussed so far, present some shortcomings; when dealing with the undesirable location problems, the following aspects have to be considered:

- it seems that the decision process for the location of an undesirable facility should consists of two stages. The first stage known as *site-generation*, would identify a small set of potential locations and the second stage, the *site-selection*, would select the final location. Many of the algorithms discussed above solve an optimization problem finding an optimal location: for example, most Euclidean 1-maximin algorithms find all local maxima for the problem. Hence, the algorithms discussed can be used as site-generating tools
- the distance measure itself must be carefully selected. For most undesirable facility problems, rectilinear distances or a tree network structure make little sense. For example, in the case of a chemical plant, it is not realistic to model the dispersal of the polluttants using a rectilinear metric. Similarly, in the case of a possible attack on several military installations, the enemy may refuse to travel along the street network, preferring to fly
- point representation of demands points and candidate sites can be an unacceptable approximation in the context of undesirable facilities. Centroids of census tracks may be appropriate when minimizing the total system cost. However, such approximations will cause inequities for people living close to the boundaries of census tracks and similarly an extended plant may not be approximated accurately by a point

- assuming convex feasible regions ignores geographical and jurisdictional realities, as well as environmental issues. County, city and state boundaries come in all shapes, and a convex feasible region makes it impossible to model many geographic barriers

- location of an undesirable facility is almost always connected with the estabilishment of an undesirable network; see for instance, high voltage power grids, hazardous waste transportation routes and so on. As well, the probability of a transport accident is related to the distance travelled. Hence, the selected objective for the location of an undesirable facility may conflict with an objective geared towards minimizing transport accident probability. A realistic model would incorporate the location and transportation objectives

- another issue that must be addressed in Location Analysis is uncertainty. Most existing methods assume that the parameters of the problem do not change through time. Especially in undesirable facility location this may be not the case; the amounts of polluttants that need to be further processed or disposed of might change significantly depending on certain technological developments, relevant legislation may also change, etc. These factors should be taken into consideration in realistic models to avoid accepting solutions that may be satisfactory today but not in the future.

  Carrizosa and Nickel [7] for instance, solve the Weber problem in the case that the demand is not known and only an estimator can be provided. Their approach consists of defining the *robustness* of a location as the maximum increase in demand needed to become infeasible, and solving an optimization problem to get the most robust location. Their method could be extended to the obnoxious case as well defininig opportunely the infeasibility of a location.

Acknowledgments

I would like to express my appreciation to Francesco Maffioli and Giorgio Gallo for valuable comments, suggestions and careful reading of the manuscript.

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